Optimizing electro-optomechanical transduction using equivalent circuits

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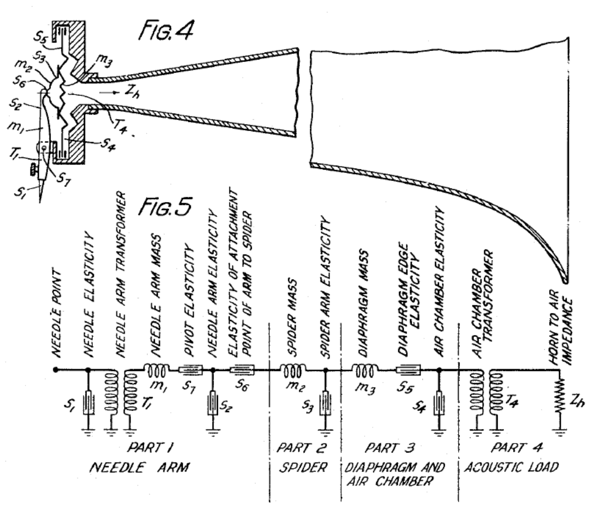
Examples from the history of equivalent circuits

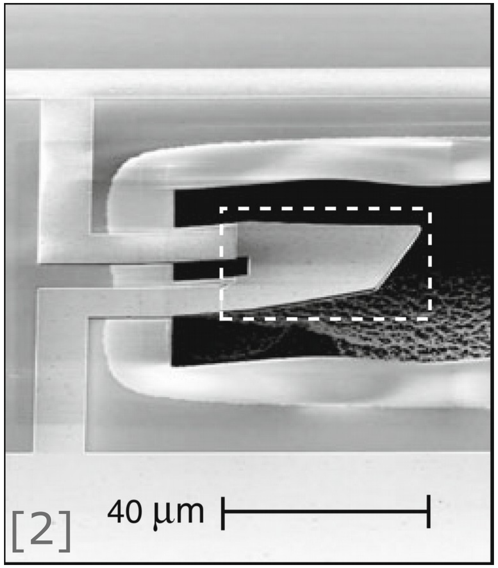
Sound reproduction



Harrison's phonograph (1923) [1]

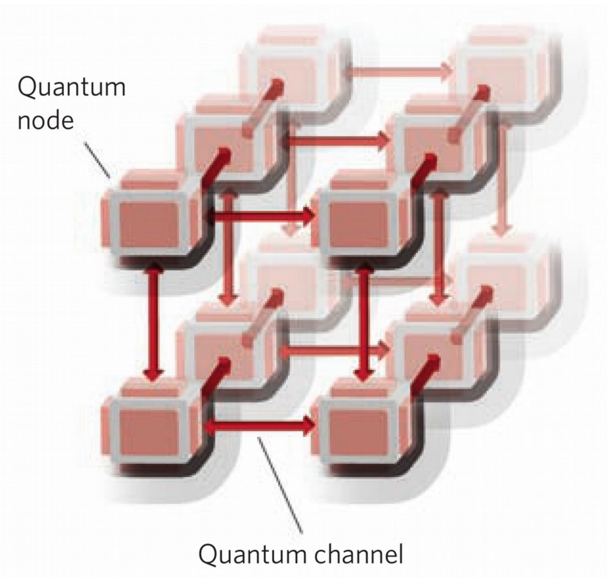
Mechanical filters in telephone multiplexing

MEMS & NEMS

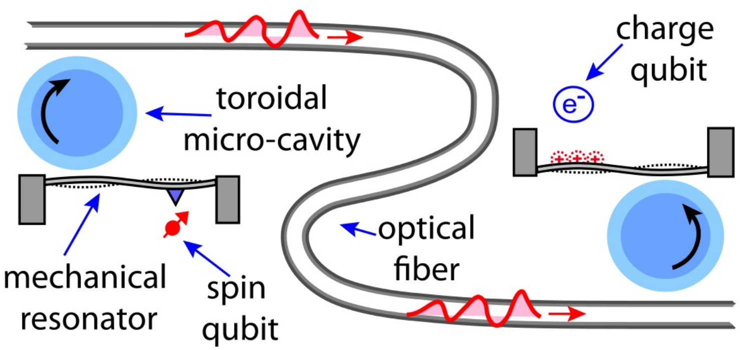


[1] US patent US1730425
[2] O'Connell & Cleland, *Cavity Optomechanics,* Springer (2014)

Quantum networks



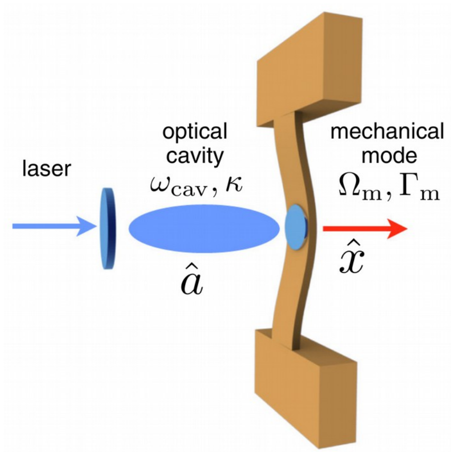
Optical internet

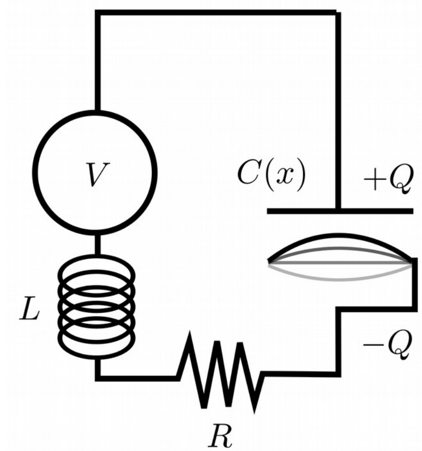


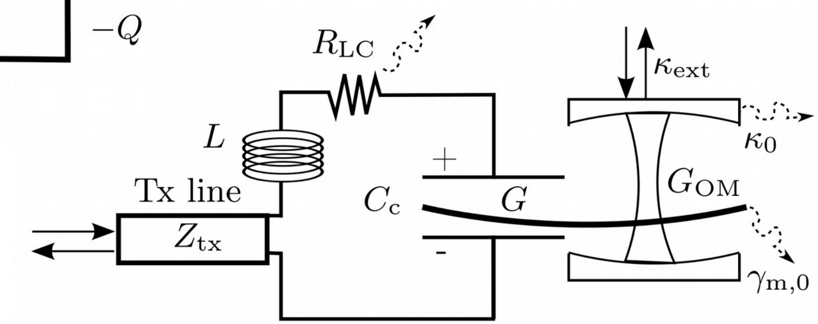
[1] Kimble; *Nature* **453**, 1023-1030 (2008)
[2] Stannigel, Rabl, Sørensen, Zoller & Lukin; *PRL* **105**, 220501 (2010)

Principles of optomechanics

Radiation pressure coupling [1]







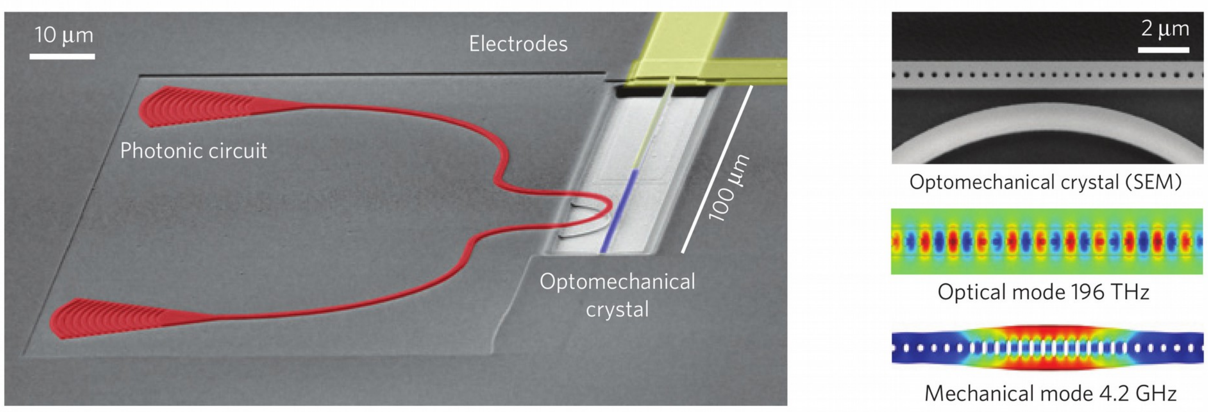
Linearized optomechanics

$$H^{(\mathrm{lin})}_{\mathrm{int,OM}} \approx \hbar g_{\mathrm{OM}}(\delta\hat{b} + \delta\hat{b}^\dagger)(e^{i\omega_1 t}\delta\hat{a} + e^{-i\omega_1 t}\delta\hat{a}^\dagger)$$

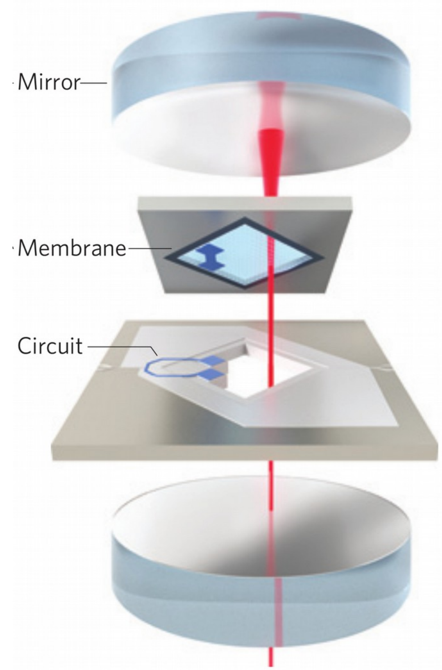
$$g_{\mathrm{OM}} \equiv \alpha \left.\frac{d\omega_{\mathrm{c}}}{dx}\right|_{x=\bar{x}} x_{\mathrm{ZPF}}$$

[1] Aspelmeyer, Kippenberg, Marquardt; *Rev. Mod. Phys.* **86**, 1391 (2014)

Electro-optomechanical transduction in experiment: Microwave-to-optical (and vice versa)



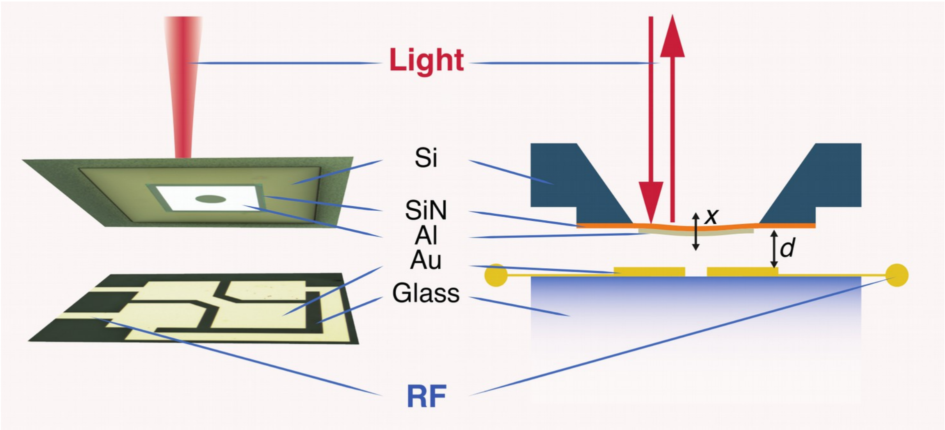
Piezoelectric optomechanical crystal [1]
Cleland's group (UCSB)



Metallized membrane-in-the-middle setup [2]
Lehnert's group (JILA)

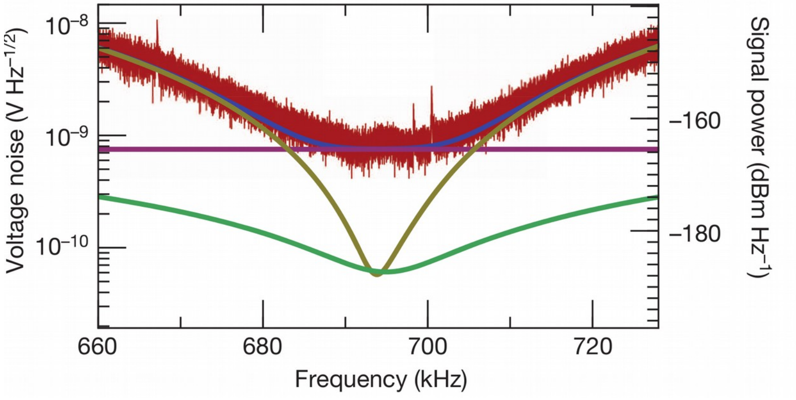
[1] Bochmann et al.; *Nature Physics* **9**, 712–716 (2013)
[2] Andrews et al.; *Nature Physics* **10**, 321–326 (2014)

Electro-optomechanical transduction in experiment: Radio-to-optical conversion



Noise budget

Electrical Optical Mech.



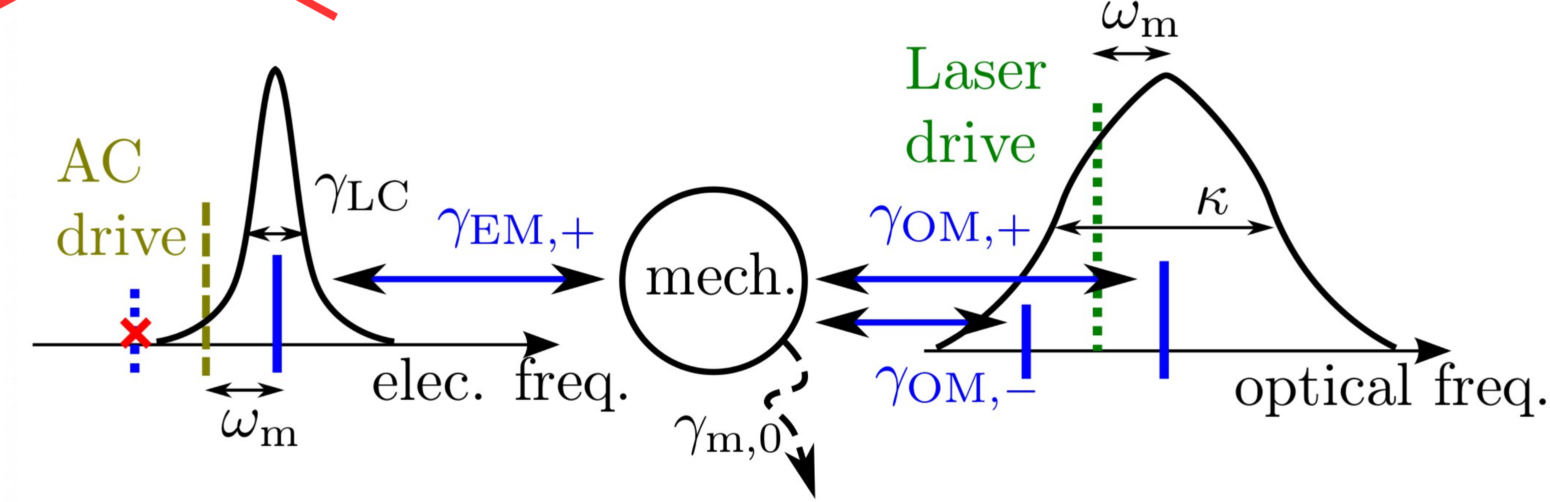
[1] Bagci, Simonsen, Schmid, Villanueva, EZ, Appel,
Taylor, Sørensen, Usami, Schliesser & Polzik; *Nature* **507**, 81–85 (2014)
[2] Schmid, Bagci, EZ, Taylor, Herring, Cassidy, Marcus, Villanueva, Amato, Boisen,
Shin, Kong, Sørensen, Usami & Polzik; *J. Appl. Phys.* **115**, 054513 (2014)

Transduction theory

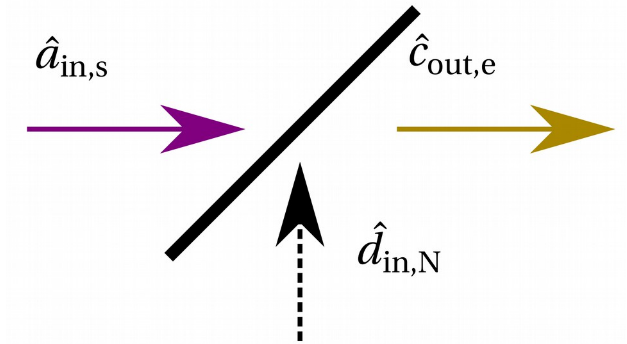
Beam splitter interaction

$$\hat{H}_{\text{int}} \sim g_1(\hat{a}+\hat{a}^\dagger)(\hat{b}+\hat{b}^\dagger) + g_2(\hat{b}+\hat{b}^\dagger)(\hat{c}+\hat{c}^\dagger)$$

$$\hat{H}_{\text{int}} \sim g_1(\hat{a}^\dagger\hat{b}+\hat{a}\hat{b}^\dagger) + g_2(\hat{b}^\dagger\hat{c}+\hat{b}\hat{c}^\dagger)$$



$$t \equiv \sqrt{\eta},\ r$$

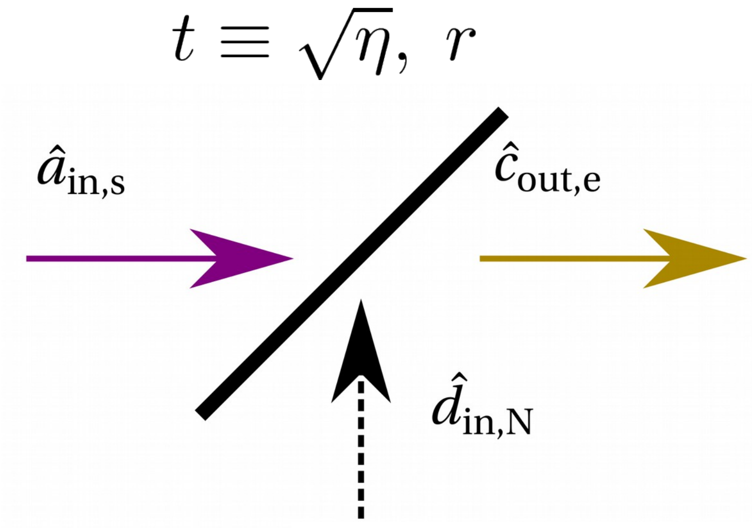


Unitarity → $1 = |t|^2 + |r|^2$

Amplification effects! [1]

[1] Carlton M. Caves; *Phys. Rev. D* **26**, 1817 (1982)

Transduction theory



Added noise (referenced to input)

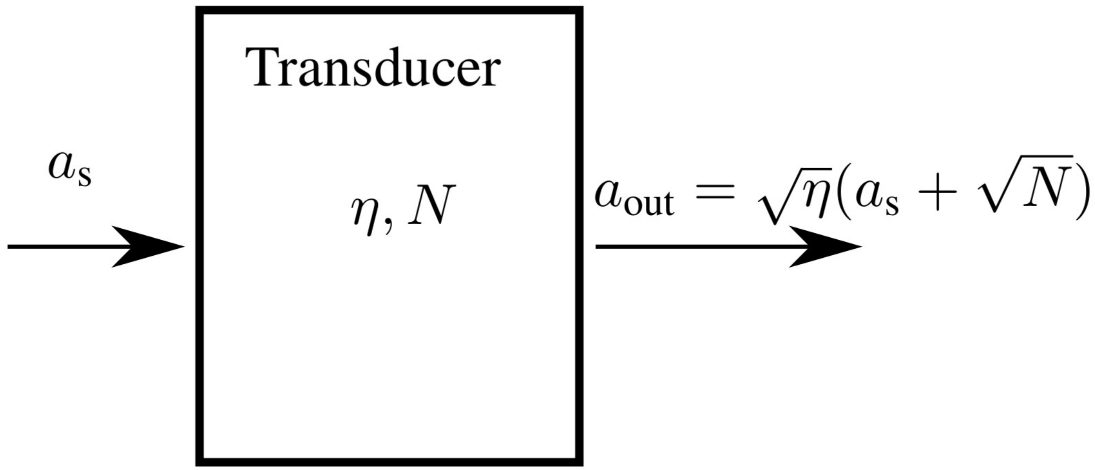
$$N(\Omega) \equiv (1/\eta) \sum_{i \neq s} |U_i(\Omega)|^2 n_i(\Omega + \omega_{\mathrm{d},i})$$

$$n_i(\omega) \approx k_{\mathrm{B}} T_i / \hbar\omega$$

Characterizing linear steady-state transducers

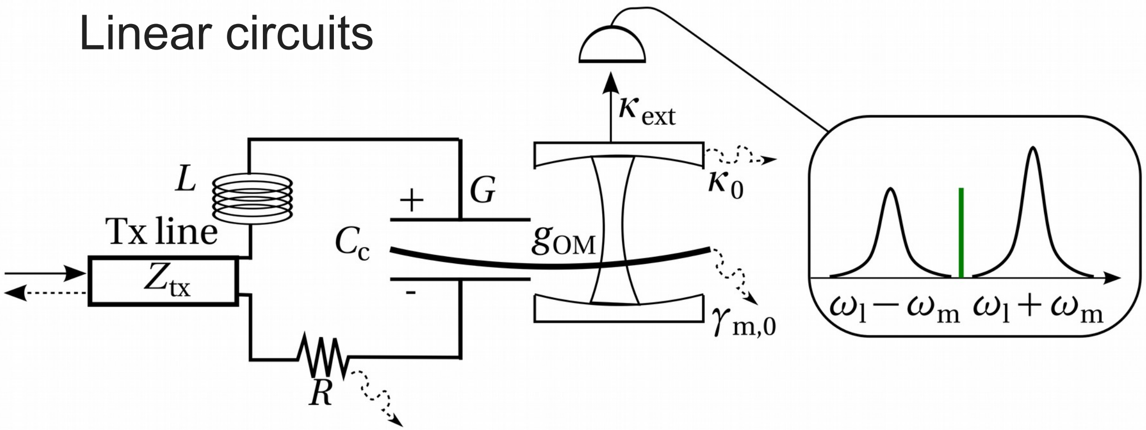
Black-box scattering description:

Signal transfer efficiency η and added noise N

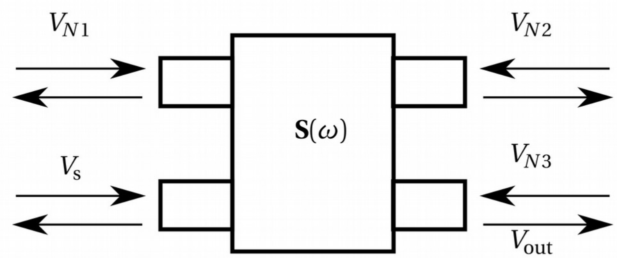


Overview of transduction scenario

Linear circuits



N-port scattering problem



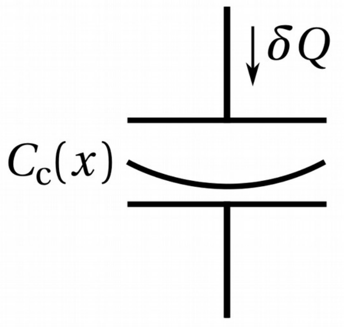
Electrical input/output theory

$$V_{-,i} = -Z_{\mathrm{tx},i} I_i + V_{+,i} \qquad \hat{V}_{\pm,i}(t) = \int_0^\infty \frac{d\omega}{\sqrt{2\pi}} \sqrt{\frac{\hbar\omega Z_{\mathrm{tx},i}}{2}} \left[\hat{b}_{\pm,i}(\omega) e^{-i\omega t} + \mathrm{H.C.}\right]$$

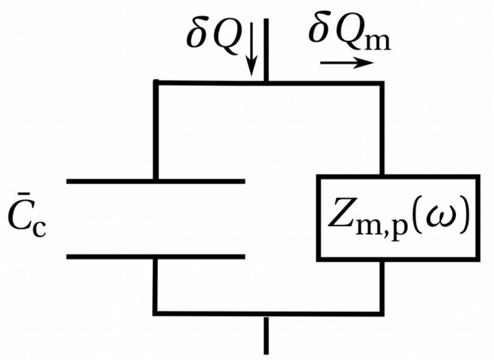
Kirchhoff's Voltage Law =

Heisenberg-Langevin eqs. of the circuit

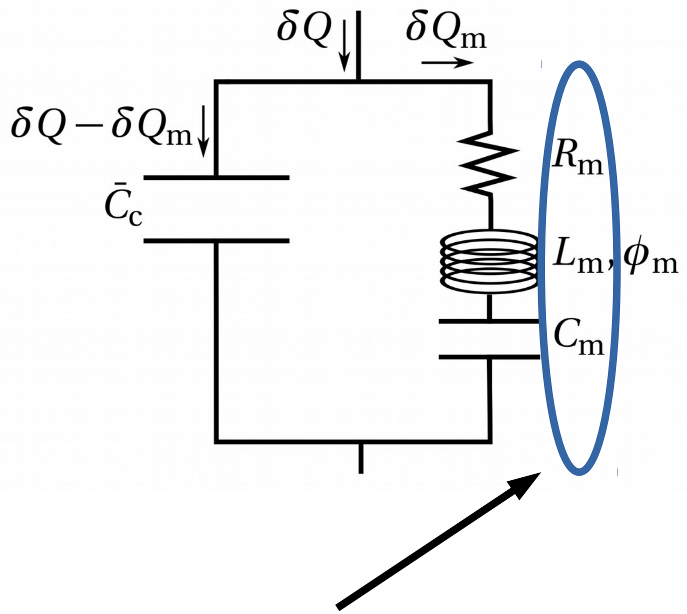
Equivalent circuit for capacitive coupling (DC bias)



$$\delta V(x) = \frac{\delta Q}{C_c(x)}$$



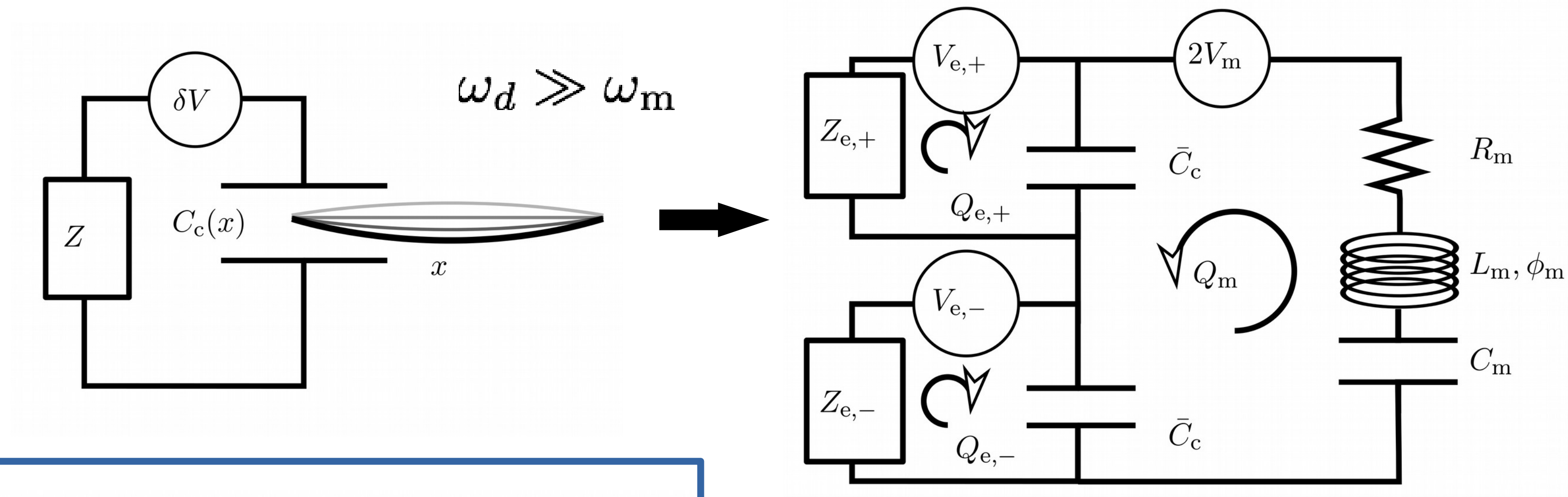
Mechanical parallel RLC arm



Mechanical equivalent circuit parameters

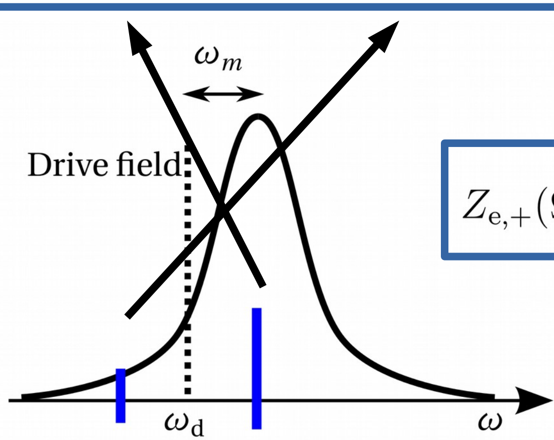
EZ, Schließer, Taylor & Sørensen; in prep.

Equivalent circuit for capacitive coupling (AC driven)



$$V_{e,+}(\Omega) = \delta V(\omega_d + \Omega) \qquad V_{e,-}(\Omega) = \delta V^*(\omega_d - \Omega)$$

$$Q_{e,+}(\Omega) = \delta Q(\omega_d + \Omega) \qquad Q_{e,-}(\Omega) = \delta Q^*(\omega_d - \Omega)$$

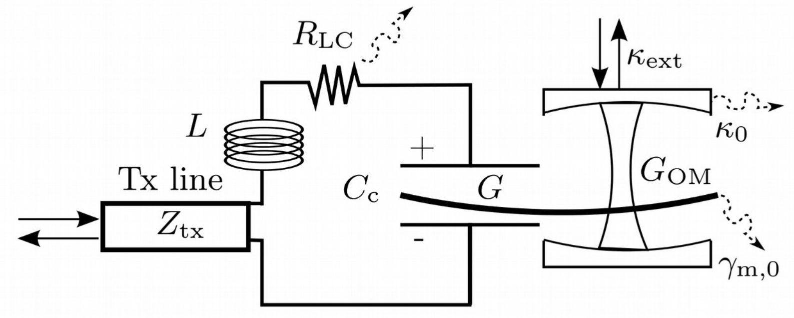


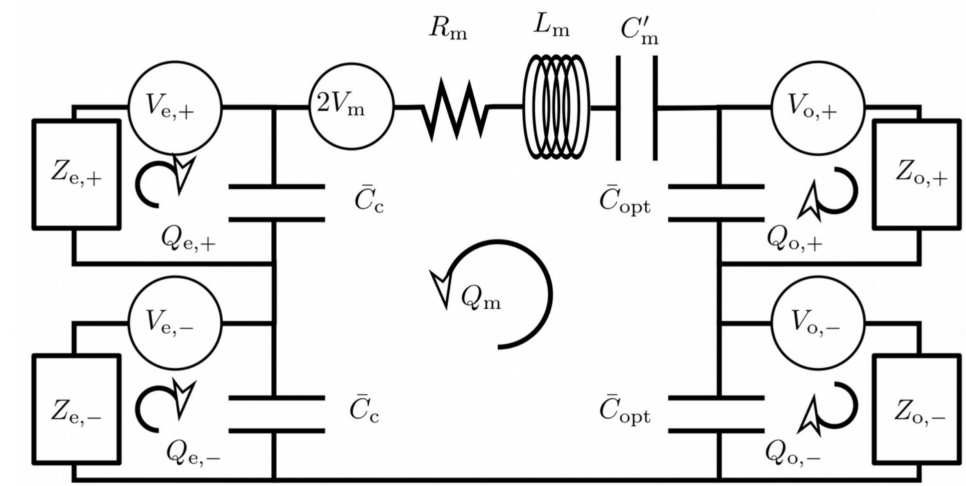
$$Z_{e,+}(\Omega) = \frac{\omega_d + \Omega}{\Omega} Z(\omega_d + \Omega) \qquad Z_{e,-}(\Omega) = -\frac{\omega_d - \Omega}{\Omega} Z^*(\omega_d - \Omega)$$

Negative resistance = amplification

EZ, Schließer, Taylor & Sørensen; in prep.

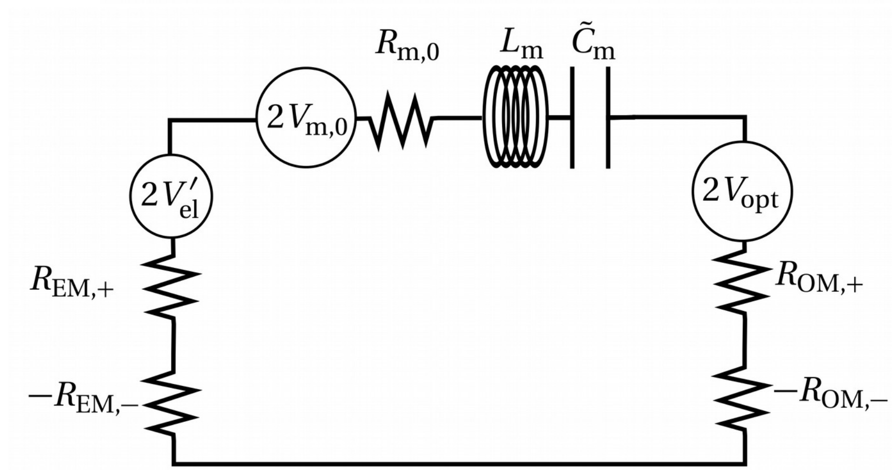
Full electro-optomechanical equivalent circuit





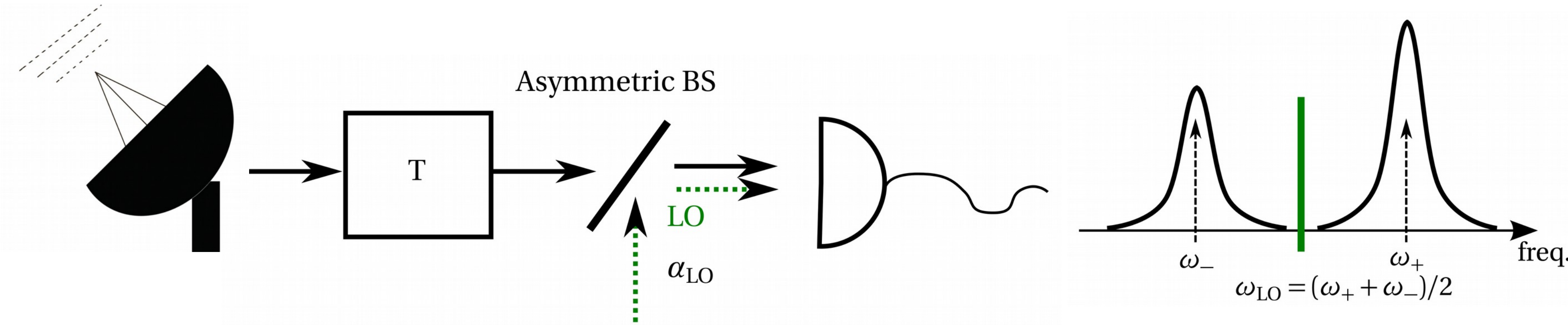
Reduced equivalent circuit

(weak coupling regime)



EZ, Schließer, Taylor & Sørensen; in prep.

Example I: Optical homodyne detection of classical RF/microwave fields

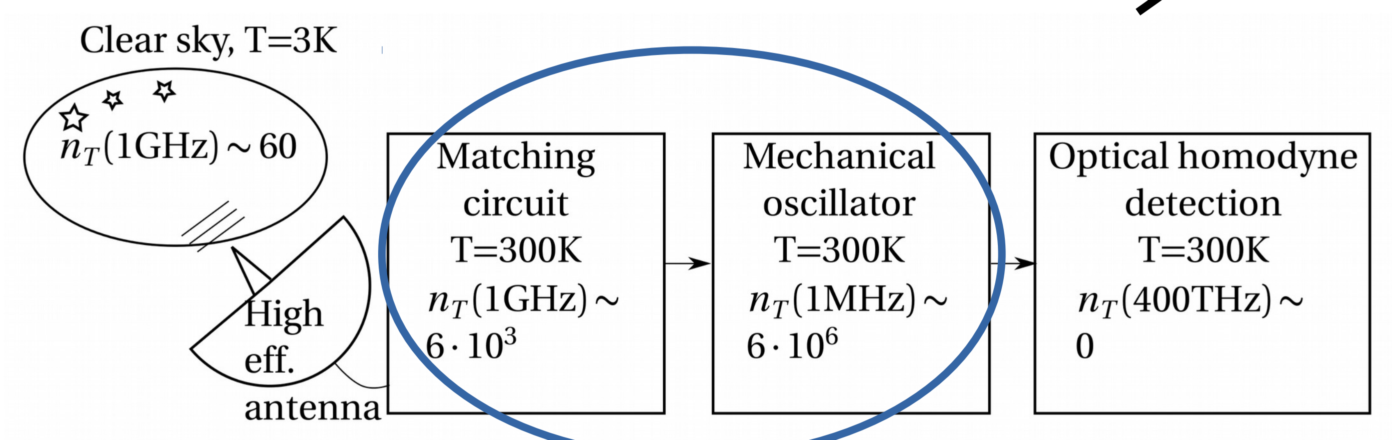


added noise

imprecision noise

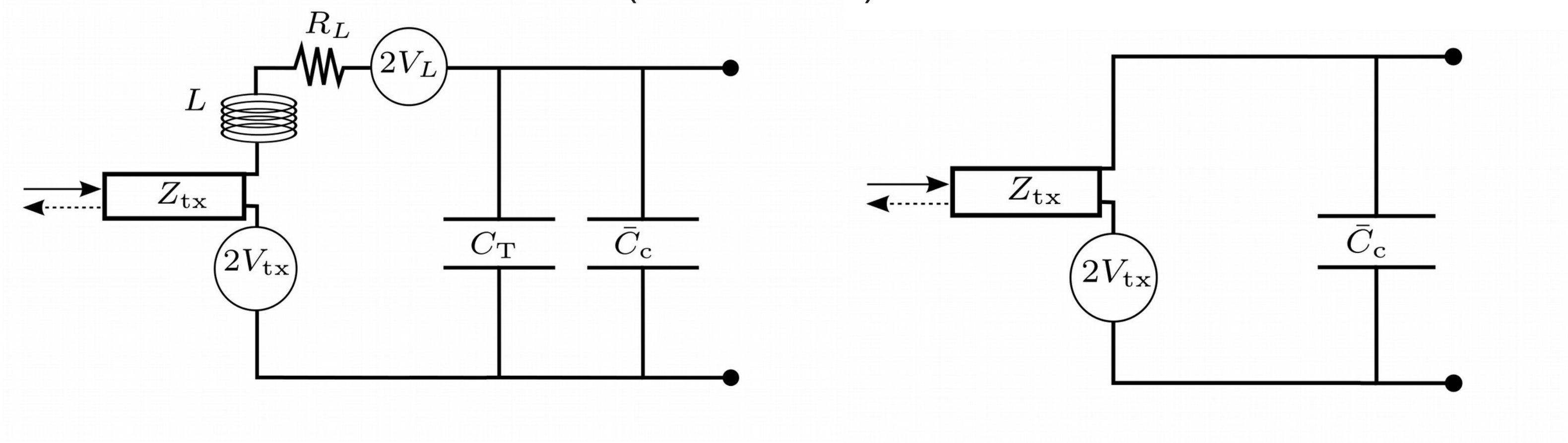
Sensitivity

$$P_s(\Omega) \sim \left(\frac{\sqrt{\eta(\Omega)}\sqrt{N(\Omega)}+\sqrt{\eta(-\Omega)}\sqrt{N(-\Omega)}}{\sqrt{\eta(\Omega)}+\sqrt{\eta(-\Omega)}}\right)^2 + \frac{1}{\left(\sqrt{\eta(\Omega)}+\sqrt{\eta(-\Omega)}\right)^2}$$

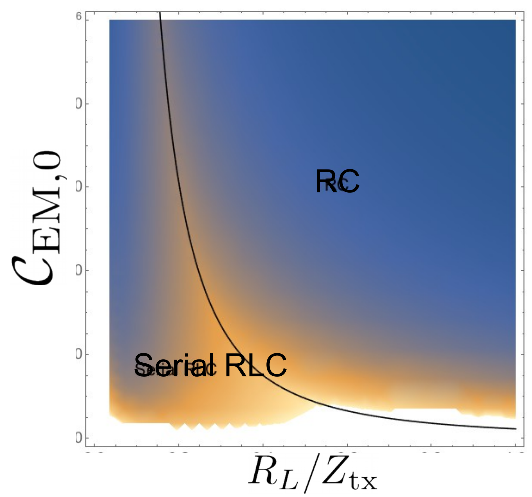


Example I:
Optical homodyne detection of classical RF/microwave fields

Receiver circuits: Serial RLC and (inductorless) RC



Minimize sum of electrical and mechanical noise contributions



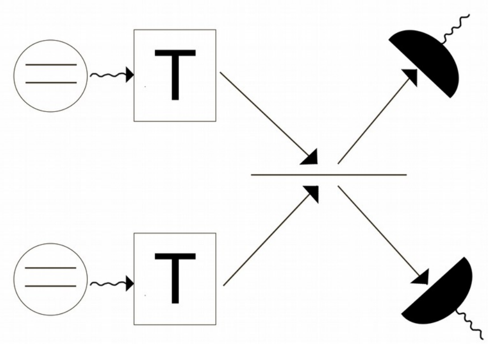
Classical, unloaded cooperativity

$$\mathcal{C}_{\mathrm{EM},0} \equiv \frac{L/\bar{C}_{\mathrm{c}}}{R_{\mathrm{m}} R_L}$$

RC adds less noise than serial RLC if:

$$\mathcal{C}_{\mathrm{EM},0} \gtrsim Q_L^2 \frac{n_{\mathrm{m}}[\omega_{\mathrm{m}}]}{n_L[\omega_{\mathrm{s}}]}$$

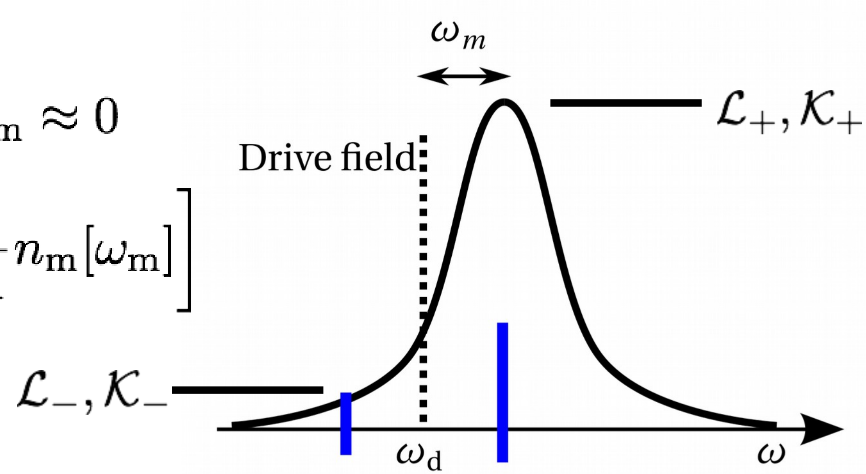
Example II: Conditional entanglement generation between superconducting qubits by optical photodetection



Conditional fidelity [2] $\eta \ll 1$

$$F_{1c} \approx 1 - 2\sqrt{N_0^{(+)} B \Delta T}$$

$$F_{2c} \approx 1 - 6 N_0^{(+)} B \Delta T$$



Electrical circuit in ground-state, $n_{\mathrm{ohm}} \approx 0$

$$N_0^{(+)} = \frac{1}{\eta_{\mathrm{el}}} \left[\frac{\mathcal{K}_-^2}{\mathcal{K}_+^2} + \frac{\gamma_{\mathrm{OM},+}}{\gamma_{\mathrm{EM},+}} \frac{\mathcal{L}_-^2}{\mathcal{L}_+^2} + \frac{\gamma_{\mathrm{m},0}}{\gamma_{\mathrm{EM},+}} n_{\mathrm{m}}[\omega_{\mathrm{m}}] \right]$$

Optimistic experimental parameters

$$\eta_{\mathrm{el}} = \eta_{\mathrm{opt}} = 0.9, \omega_{\mathrm{el}}/(2\pi) \sim 7\mathrm{GHz}, \omega_{\mathrm{m}}/(2\pi) \sim 1\mathrm{MHz}, B\Delta T = 3$$

$$\kappa = \gamma_{\mathrm{el}} = 0.4\omega_{\mathrm{m}}, \mathcal{C}_{\mathrm{EM}} = \mathcal{C}_{\mathrm{OM}} = 10^4, T = 10\mathrm{mK} \Rightarrow \boxed{F_{2c} \sim 0.76}$$

[1] Möhring, et al.; *J. Opt. Soc. Am. B* **24**, 300-315 (2007)
[2] EZ, Schließer, Sørensen & Taylor; in prep.

Conclusions

Transduction is non-trivial to optimize

- Requires application-specific optimization of
  signal transfer efficiency η, and
  added noise N
- Impedance matching is not optimal in general

Electro-optomechanical equivalent circuits

- Puts optomechanics in the electrical engineering toolbox
- Convenient method for extracting physics of interest

Outlook

Real-world applications

- Nuclear Magnetic Resonance Imaging (NMRI)
- Radio-astronomy

Combine with quantum theory of non-linear circuits [1,2]

[1] Burkhard et al., PRB **69**, 064503 (2004); Burkhard, PRB **71,** 144511 (2005)
[2] Nigg et al., PRL **108**, 240502 (2012)