Topological energy transfer in an optomechanical system with exceptional points

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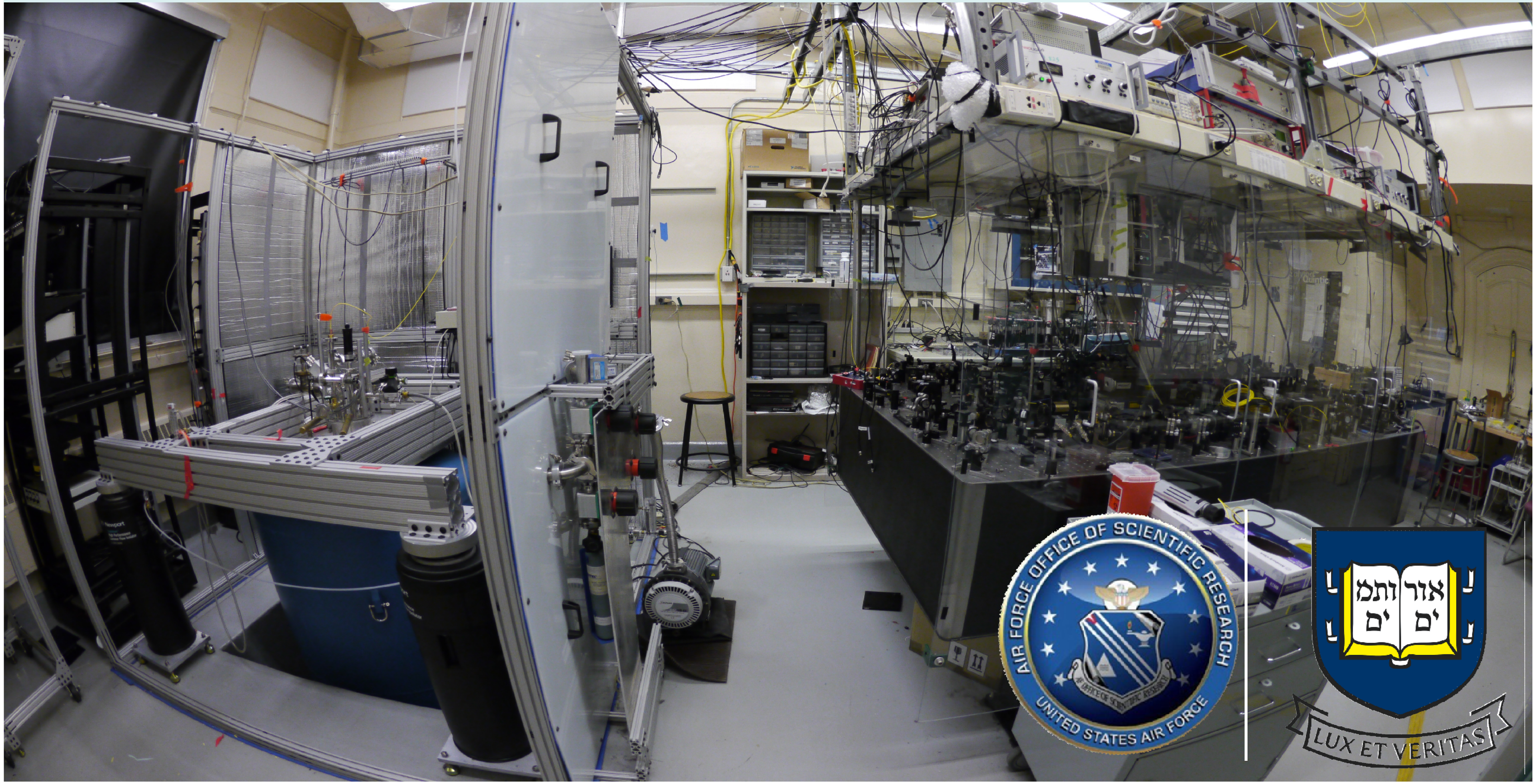


Topology

Topology: property preserved under continuous deformation

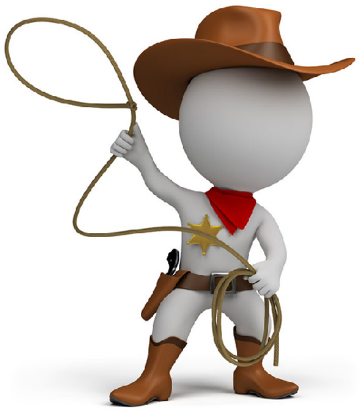
$\neq$

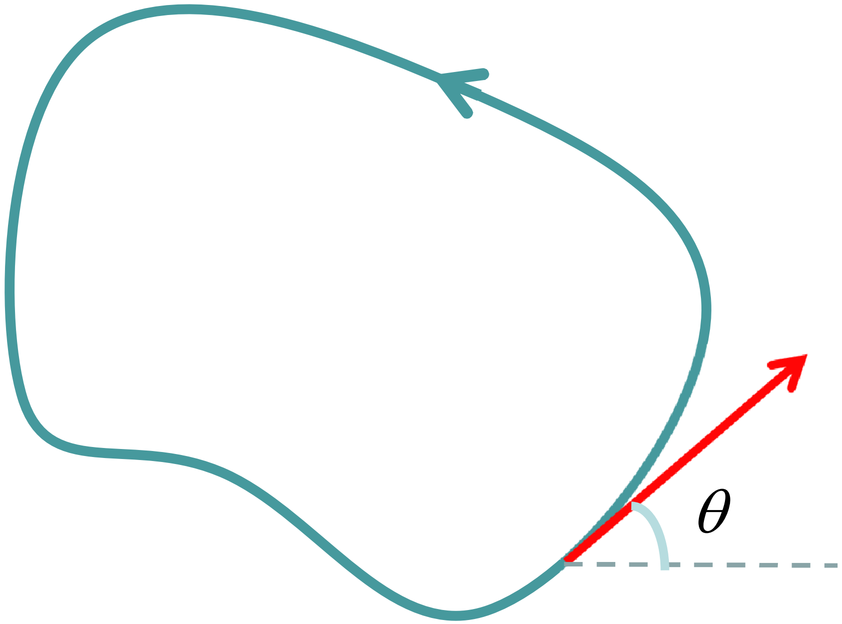
$$\Big\| \quad \oint\!\!\!\oint \kappa \, dA = 4\pi(1-g) \quad \Big\|$$

$\neq$

Topology

Loop:

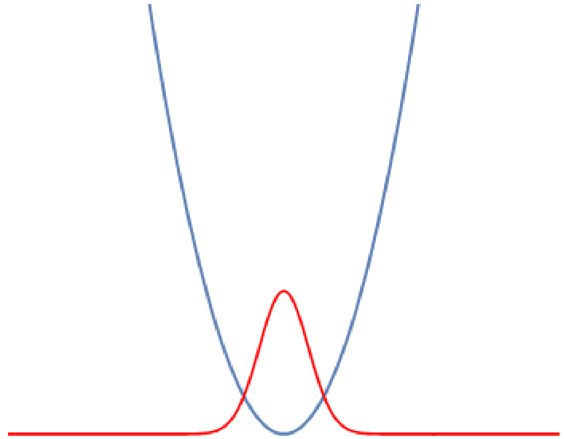




$$\oint \kappa dl = \oint d\theta = 2\pi \times n$$

Topological physics arising from adiabatic evolution

Adiabatic theorem:



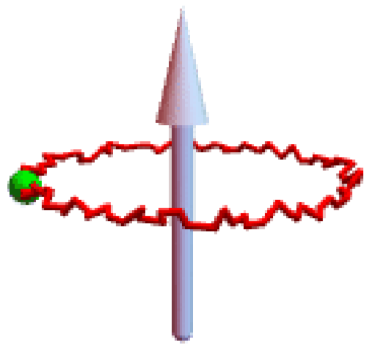
$$|\Psi_n(t)\rangle = e^{i\gamma_n(t)} e^{-\frac{i}{\hbar}\int_0^t dt'\varepsilon_n(R(t'))} |n(R(t))\rangle$$

Berry Phase:

$$\gamma_n(t) = i\int_{R(0)}^{R(t)} dR \cdot \underbrace{\langle n(R)|\nabla_R n(R)\rangle}_{\text{Berry connection}}$$

Berry phase from topological operations:

Aharanov-Bohm effect:



$$\gamma_n(C) = \frac{e}{\hbar c}\Phi_B$$

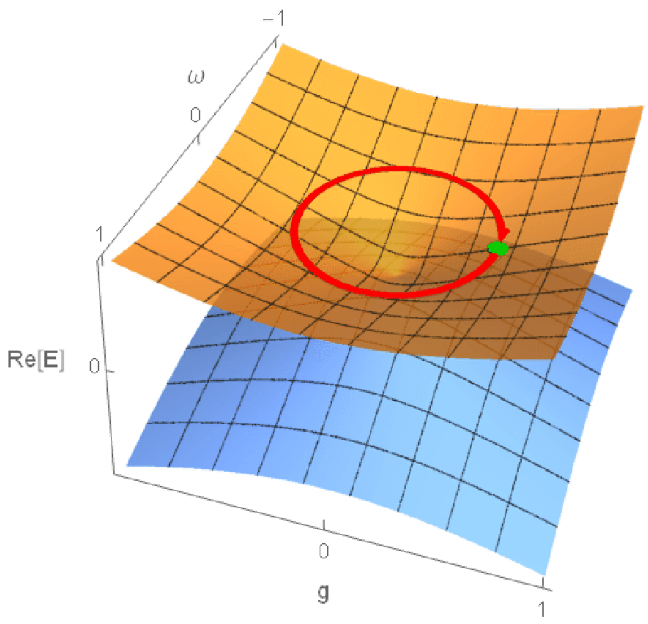
Energy transfer by topological operations

Here we realize energy transfer by topological operations.

- Fundamental interest (Heiss 1999, Keck2003, Berry 2004, Milburn 2015)
- Robustness to perturbations and inaccuracy in system control
- Non-Reciprocity (Berry 2011, Uzdin 2011, Gilary 2012, Thilagam 2012, Gilary 2013, Milburn 2015)
- Potential applications:
  Vibrational cooling of molecules and population transfer/purification (Atabek 2011, Lefebvre 2011, Hamamda 2015),
  Atomic switch and logic operations (Jaouadi 2013, Petra 2014)
  Topological quantum computation (Kim 2013).

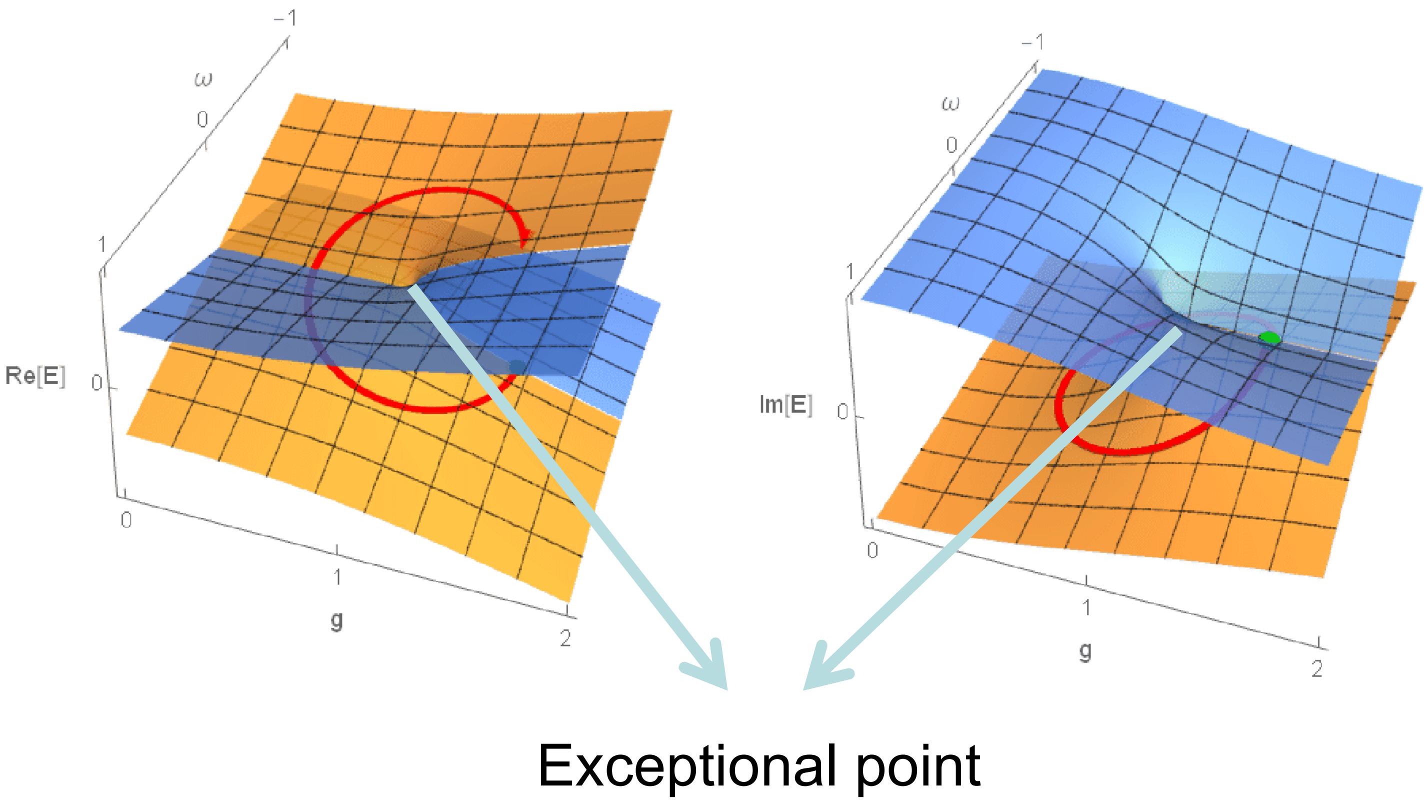
Toy model

$$H = \begin{pmatrix} \omega & g \\ g & -\omega \end{pmatrix}$$



Toy model

$$H = \begin{pmatrix} \omega + i & g \\ g & -\omega - i \end{pmatrix}$$

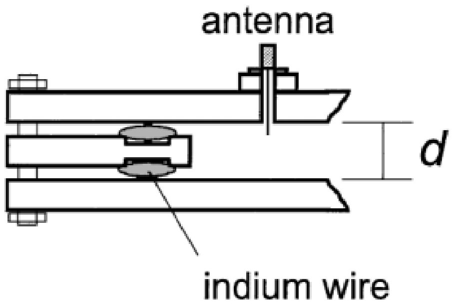


Exceptional points

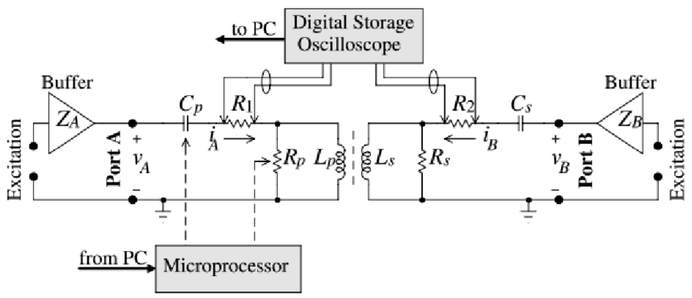
EP:

A topological singularity in the complex spectrum of an open system where two eigenvalues coalesce (Kato1966).

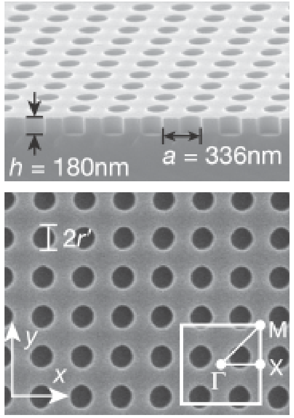
EPs exist in various systems:



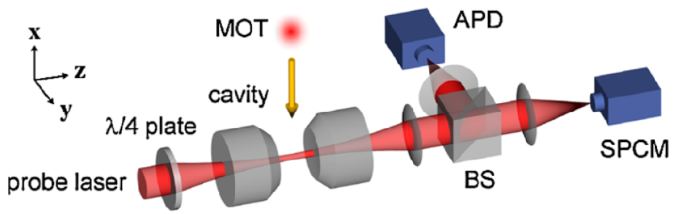
Dembowski, PRL 86, 787 (2001)



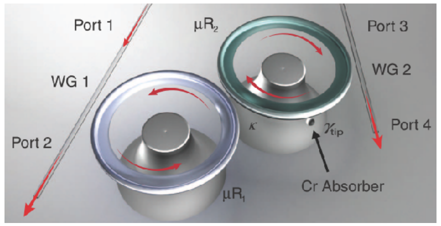
Stehmann, JPA 37, 7813 (2004)



Zhen, Nature 525, 354 (2015)



Choi, PRL 104, 153601 (2010)

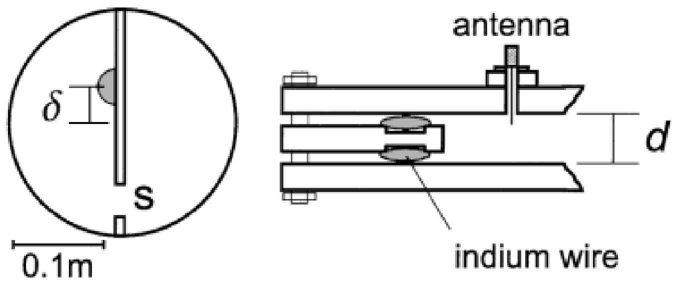


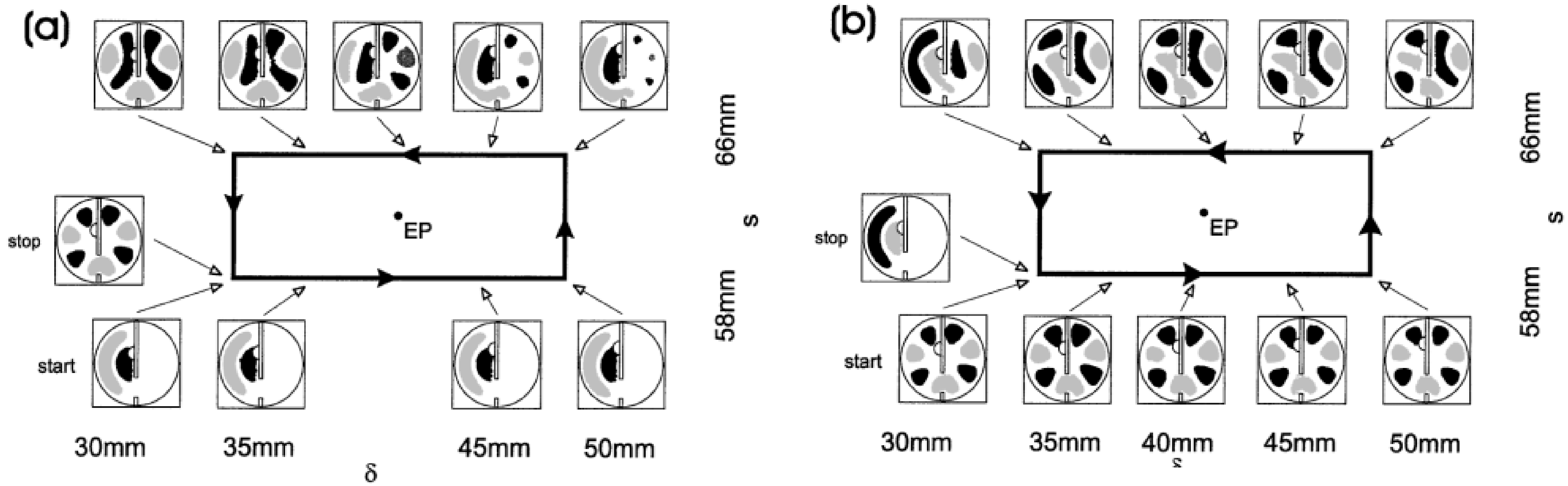
Peng, Science 346, 328 (2014)

Earlier experiments

Static measurements of eigenmodes around EP:

Microwave cavity (Dembowski PRL 2001)

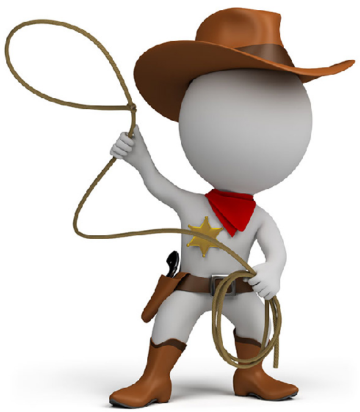


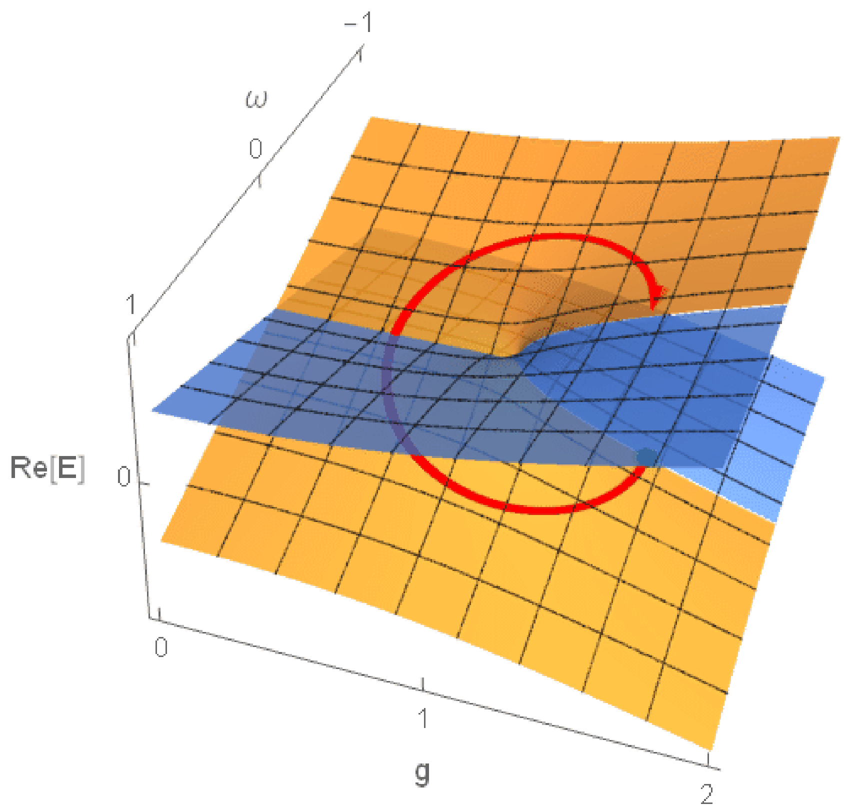


Earlier experiments

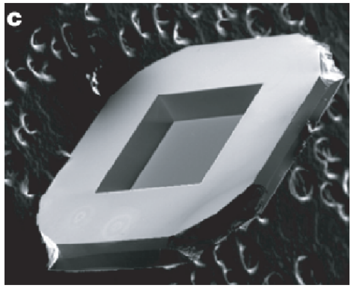
Static measurements of eigenmodes around EP:

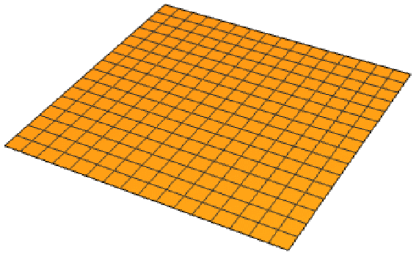
Topological dynamics?

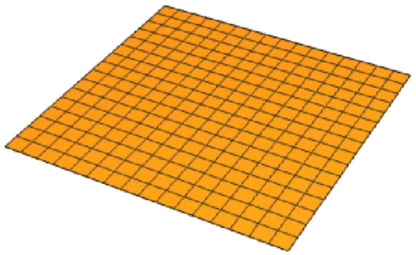




Membrane-in-the-middle optomechanical system







Almost perfectly square membrane suspended over a silicon frame

Dimension 1 mm X 1 mm X 50 nm

Focus on two modes ((1,3) and (3,1)) with bare frequencies 788.024 kHz and 788.487 kHz.

Well separated from other mechanical modes.

Bare damping rates 0.6 Hz and 1.4 Hz.

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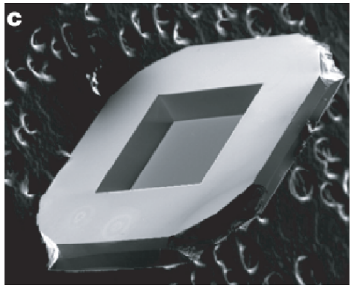

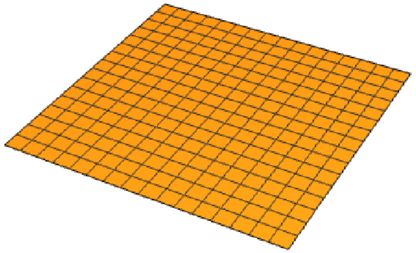

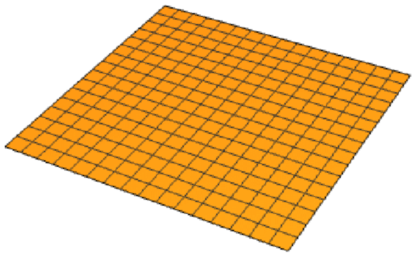

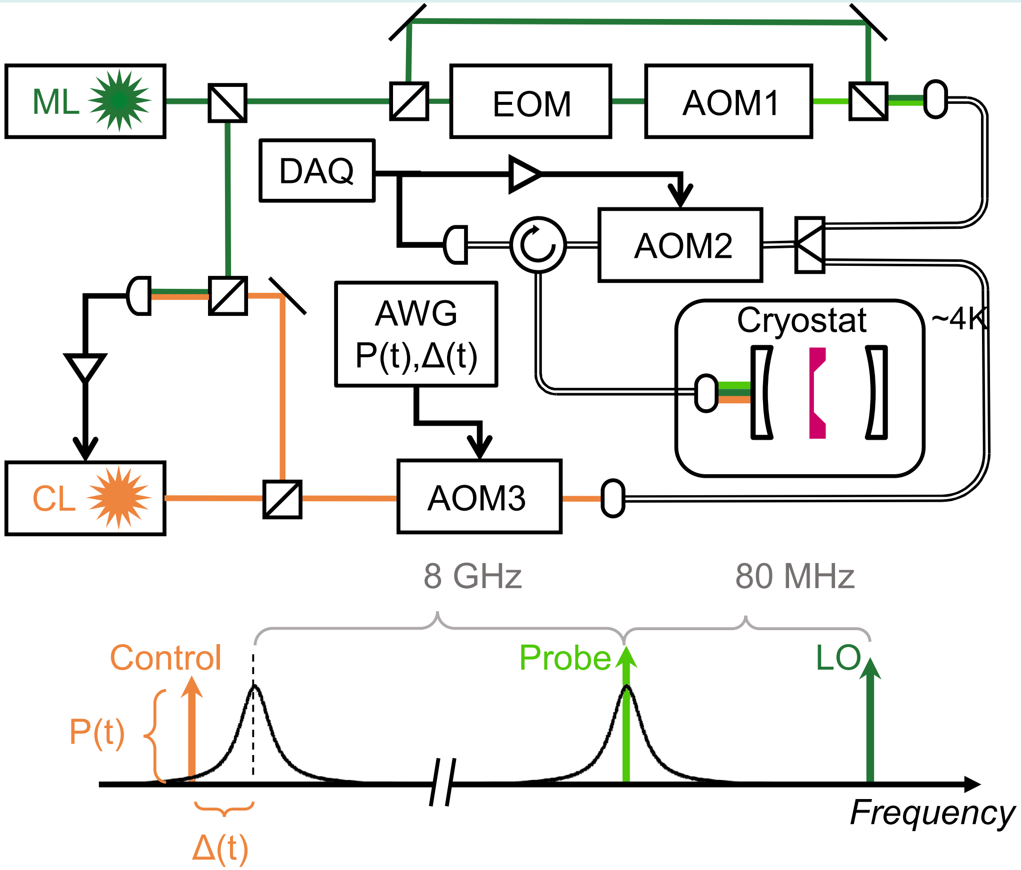

The membrane motion of one mode modulates the intracavity field, which drives the other mechanical mode, and vice versa.

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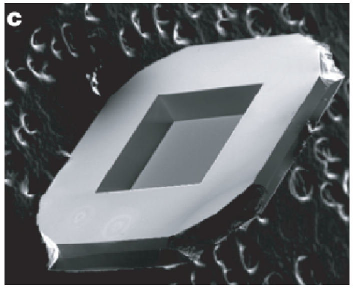
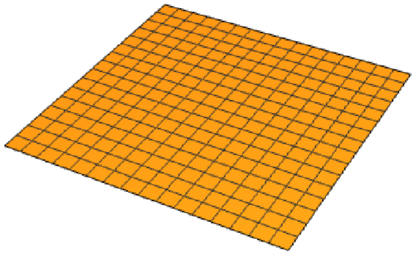
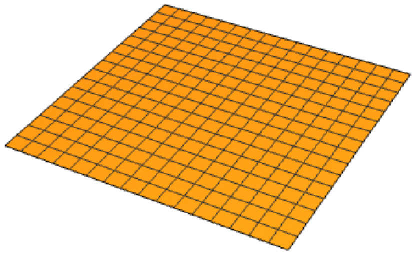






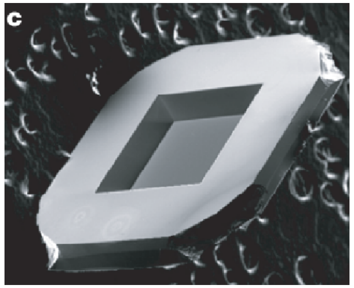
Equation of motion:

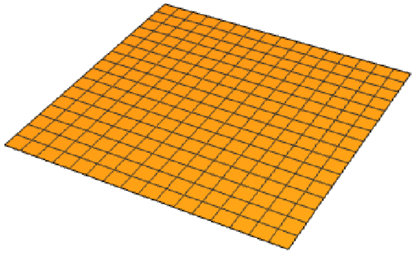
$$\dot{a} = -\left(\frac{\kappa}{2} + i\omega_c\right)a - ig_1 a z_1 - ig_2 a z_2 + \sqrt{\kappa_{in}}\, a_{in}$$

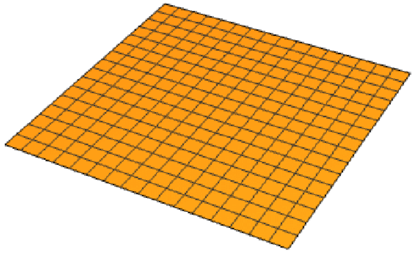
$$\dot{c}_1 = -\left(\frac{\gamma_1}{2} + i\omega_1\right)c_1 - ig_1 a^* a + \sqrt{\gamma_1}\,\eta_1$$

$$\dot{c}_2 = -\left(\frac{\gamma_2}{2} + i\omega_2\right)c_2 - ig_2 a^* a + \sqrt{\gamma_2}\,\eta_2$$

Membrane-in-the-middle optomechanical system







Equation of motion:

$$\dot{a} = -\left(\frac{\kappa}{2} + i\omega_c\right)a - ig_1 a z_1 - ig_2 a z_2 + \sqrt{\kappa_{\rm in}}\, a_{\rm in}$$

$$\dot{c}_1 = -\left(\frac{\gamma_1}{2} + i\omega_1\right)c_1 - ig_1 a^* a + \sqrt{\gamma_1}\,\eta_1$$

$$\dot{c}_2 = -\left(\frac{\gamma_2}{2} + i\omega_2\right)c_2 - ig_2 a^* a + \sqrt{\gamma_2}\,\eta_2$$

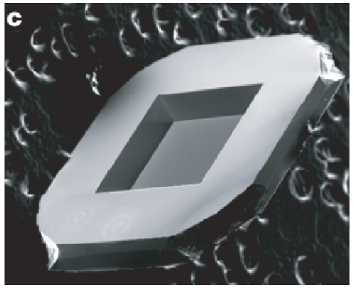
Integrating out the optical field:

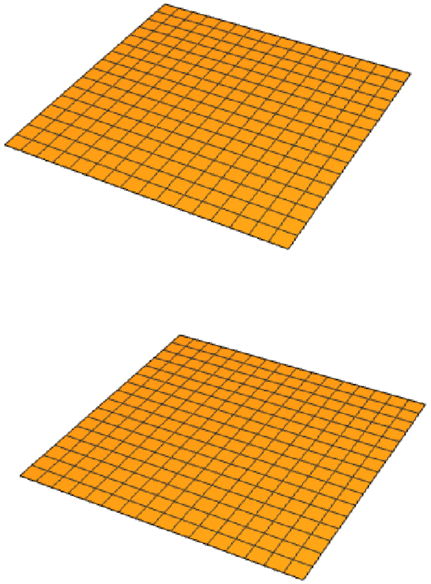
$$i\dot{\mathbf{C}}(t) = H\mathbf{C}(t)$$

$$H = \begin{pmatrix} \omega_1 - i\gamma_1/2 - ig_1{}^2\Sigma & -ig_1 g_2\Sigma \\ -ig_1 g_2\Sigma & \omega_2 - i\gamma_2/2 - ig_2{}^2\Sigma \end{pmatrix}$$

$$\Sigma = \frac{P}{\hbar\Omega_L}\frac{\kappa_{in}}{(\kappa/2)^2 + \Delta^2}\left(\frac{1}{\kappa/2 - i(\omega_0 + \Delta)} - \frac{1}{\kappa/2 + i(-\omega_0 + \Delta)}\right)$$

Membrane-in-the-middle optomechanical system



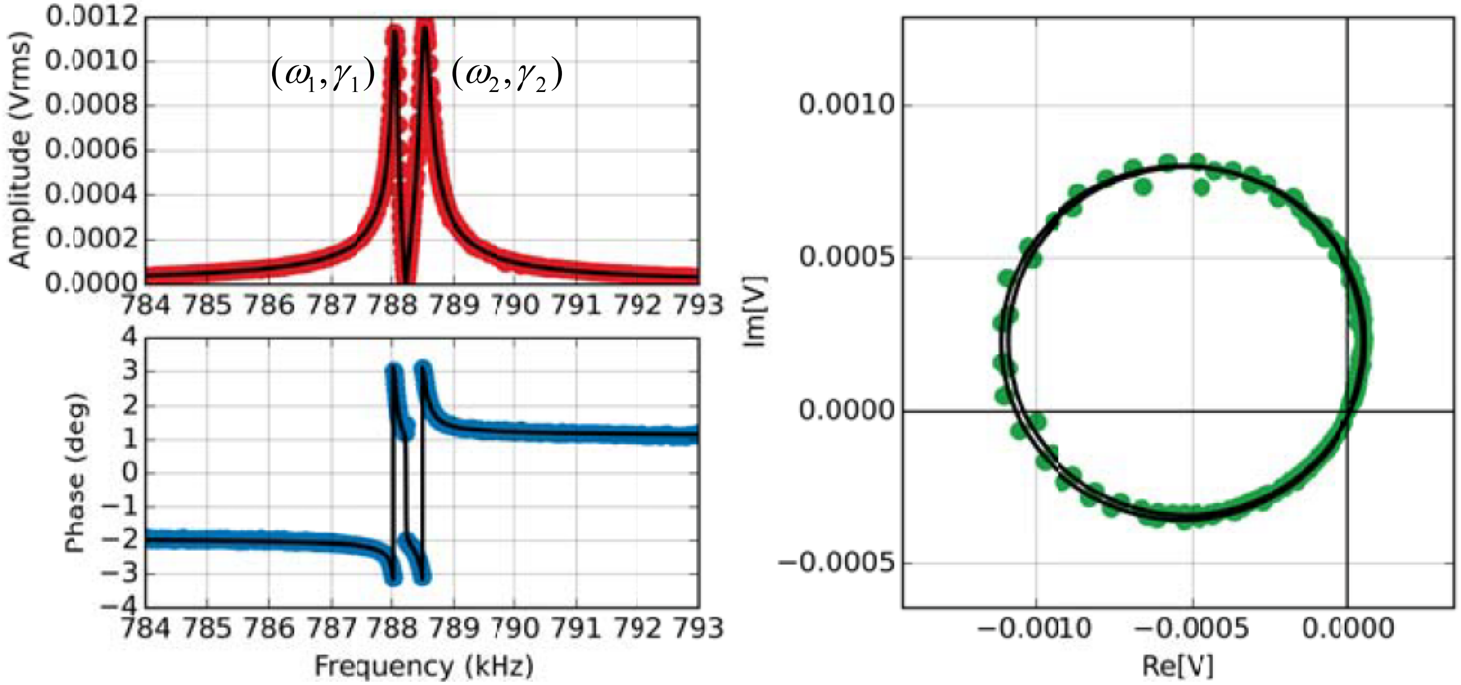


Condition for EP:

$$(\omega_1 - i\gamma_1/2 - \omega_2 + i\gamma_2/2)\left[-i\left(g_1^2 - g_2^2\right) \pm 2g_1g_2\right]/\left(g_1^2 + g_2^2\right)^2$$

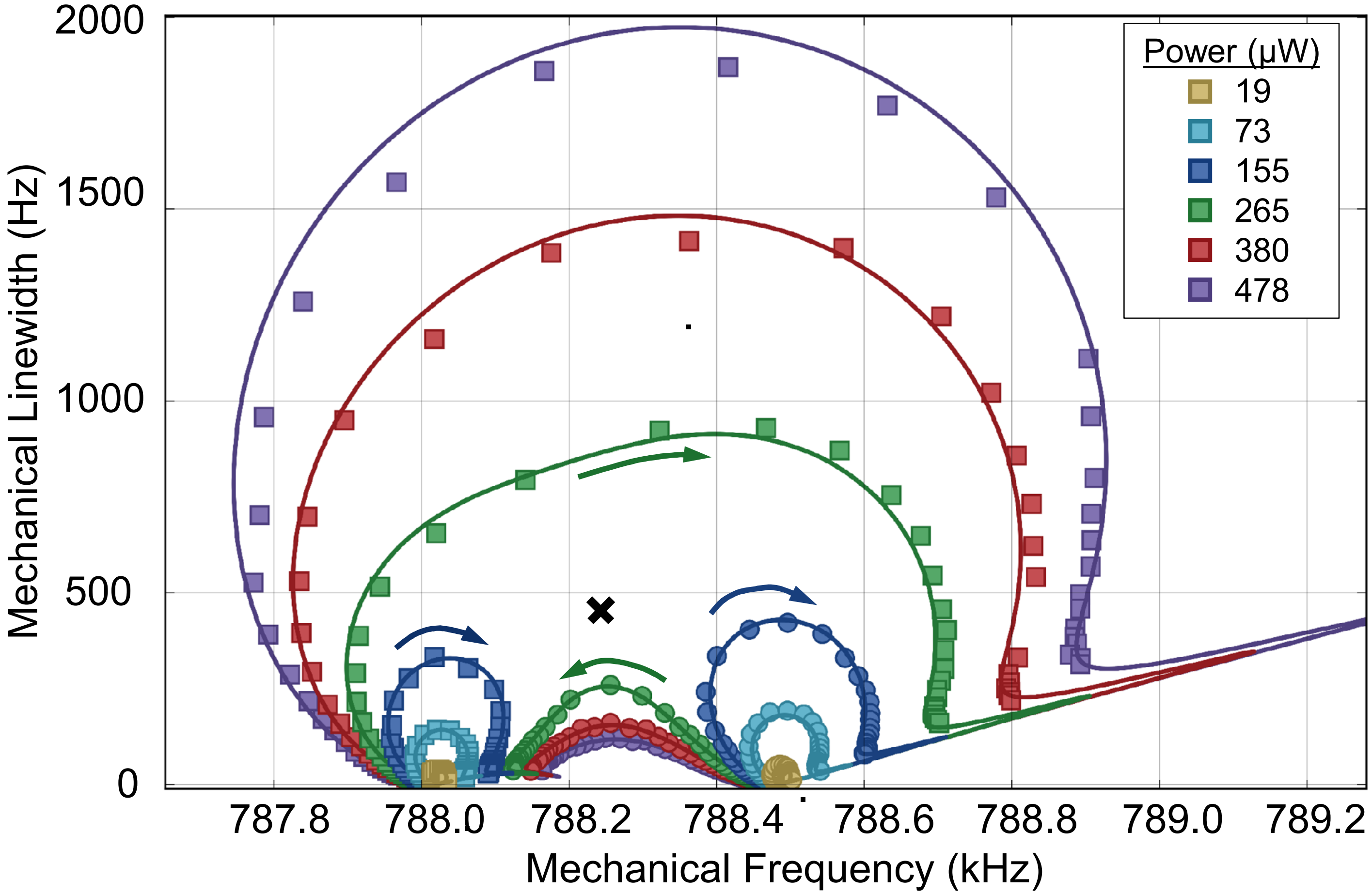
$$= \frac{P}{\hbar\Omega_L}\frac{\kappa_{in}}{(\kappa/2)^2 + \Delta^2}\left(\frac{1}{\kappa/2 - i(\omega_0 + \Delta)} - \frac{1}{\kappa/2 + i(-\omega_0 + \Delta)}\right)$$

Spectrum



Measurement of the complex eigenvalue.
(Fix power and detuning, sweep the external drive frequency.)

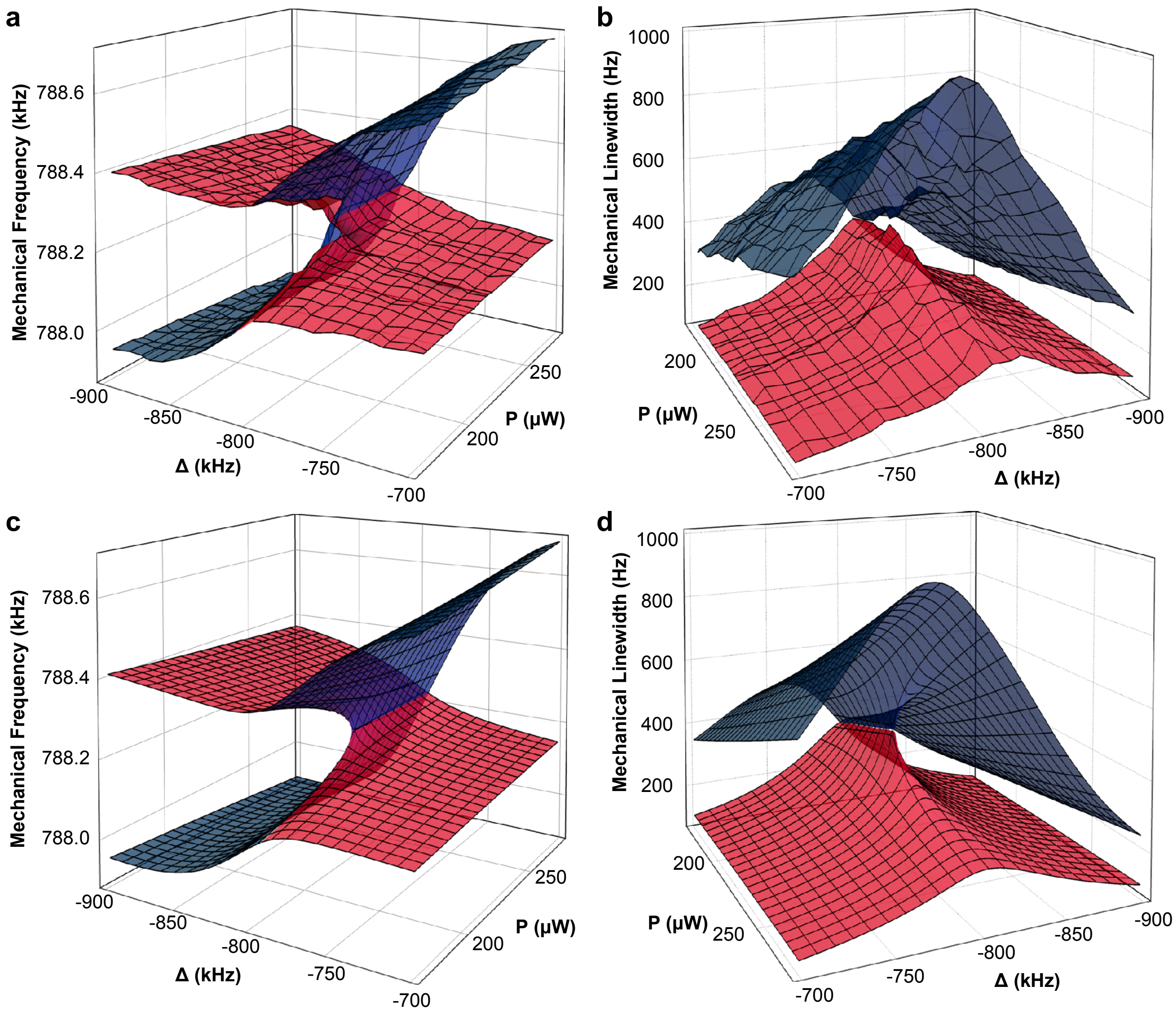
Spectrum



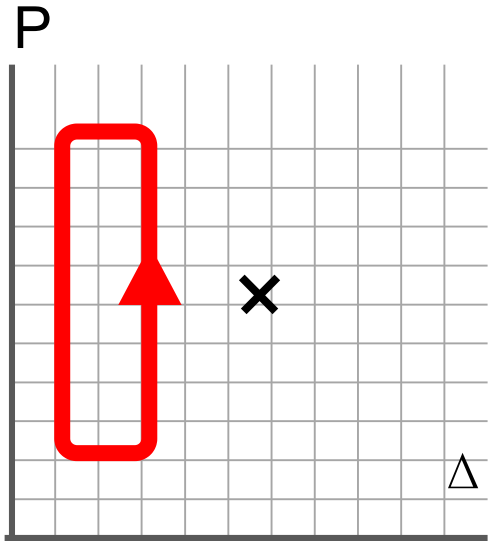
$g_1/(2\pi) = 1.03\,\mathrm{Hz}$ $\quad\kappa_{in}/(2\pi) = 70\,\mathrm{kHz}$

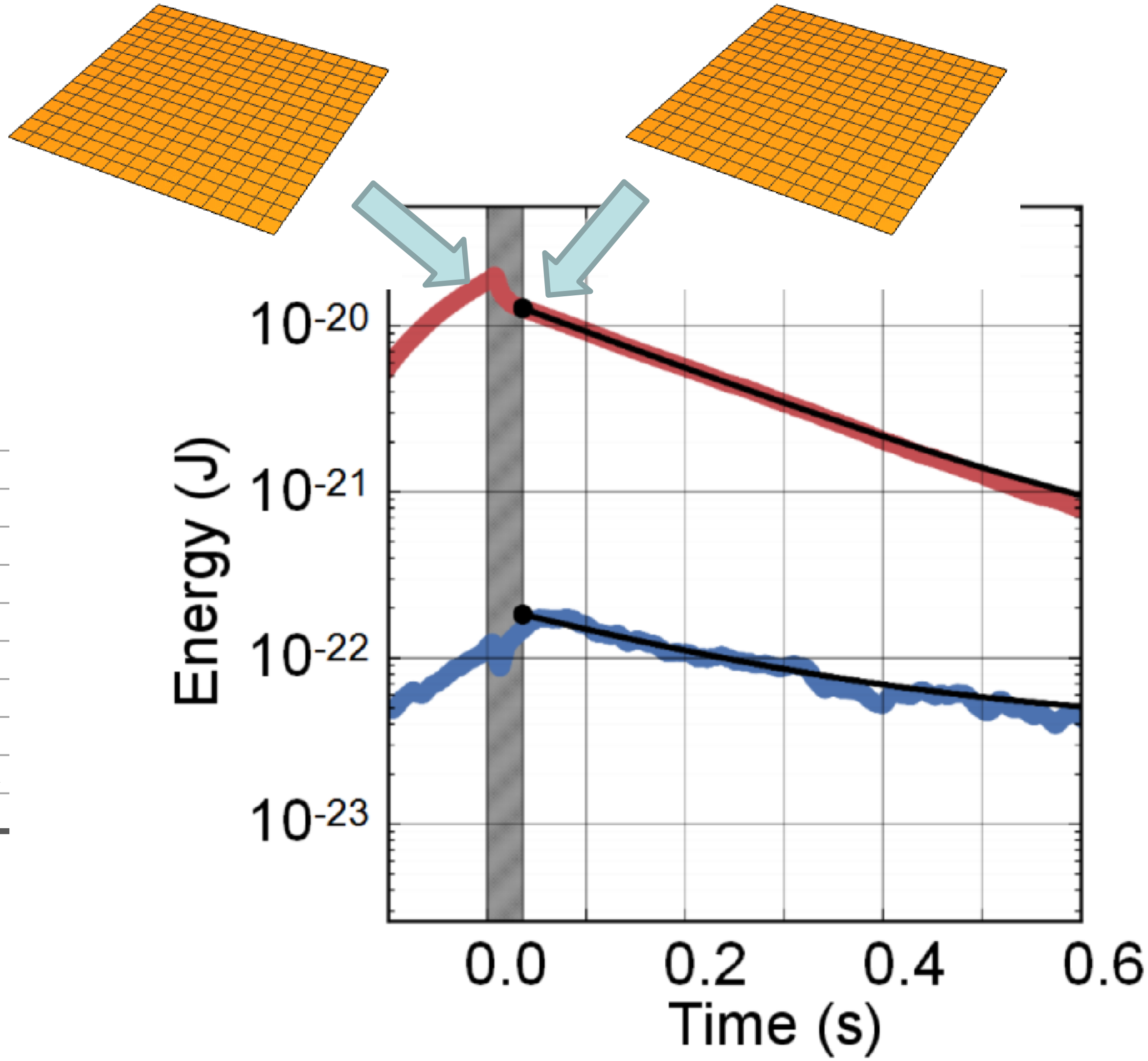
$g_2/(2\pi) = 1.14\,\mathrm{Hz}$ $\quad\kappa/(2\pi) = 177\,\mathrm{kHz}$

Spectrum


a
Mechanical Frequency (kHz)
788.6
788.4
788.2
788.0
-900
-850
-800
-750
-700
Δ (kHz)
P (μW)
200
250
b
Mechanical Linewidth (Hz)
1000
800
600
400
200
P (μW)
200
250
-700
-750
-800
-850
-900
Δ (kHz)
c
Mechanical Frequency (kHz)
788.6
788.4
788.2
788.0
-900
-850
-800
-750
-700
Δ (kHz)
P (μW)
200
250
d
Mechanical Linewidth (Hz)
1000
800
600
400
200
P (μW)
200
250
-700
-750
-800
-850
-900
Δ (kHz)

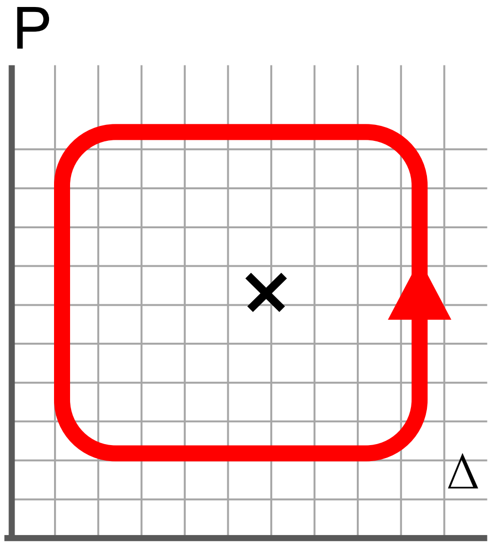
Topological Energy Transfer

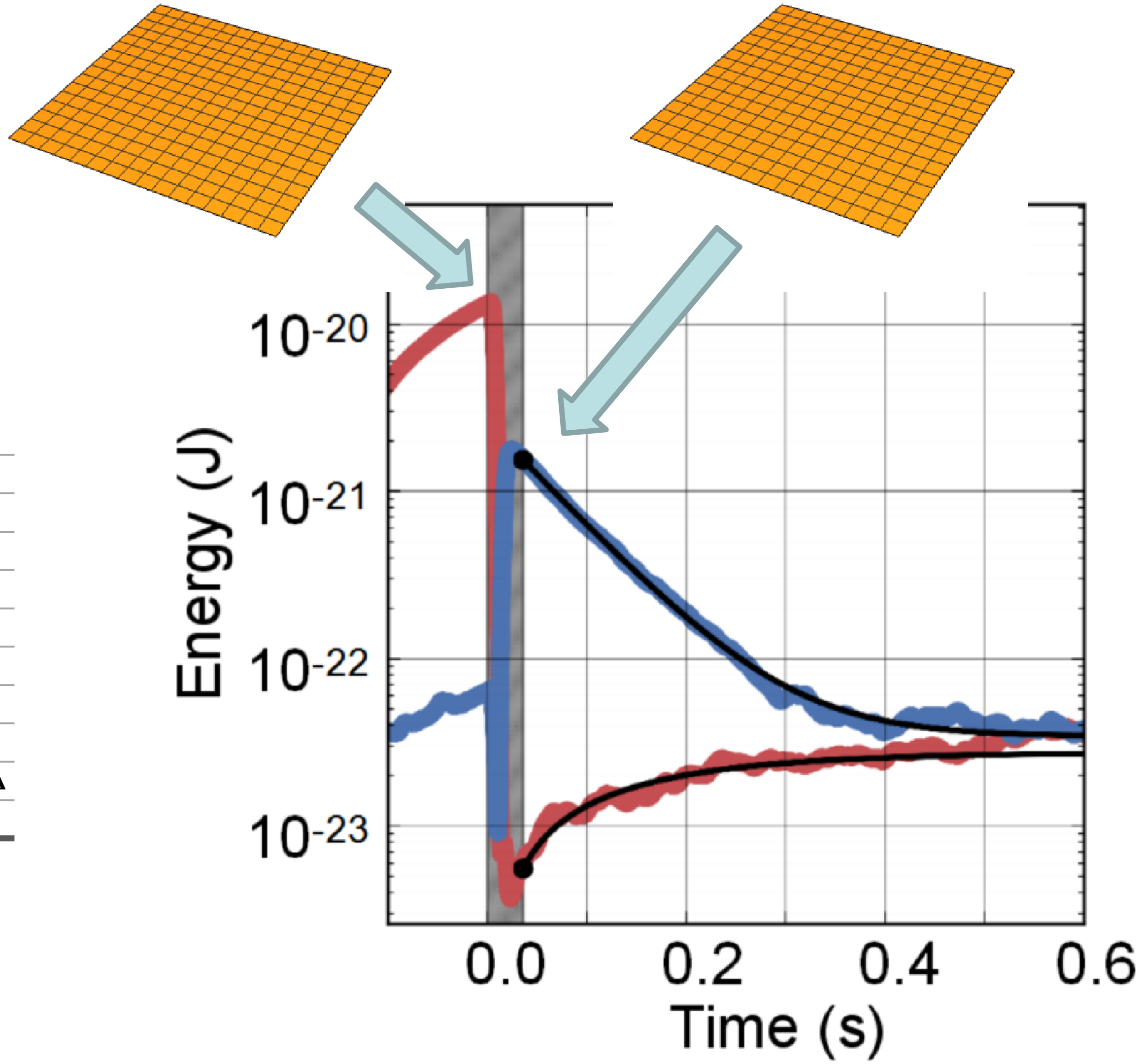




Dependence of the system's adiabatic dynamics on
the topology of the control loop

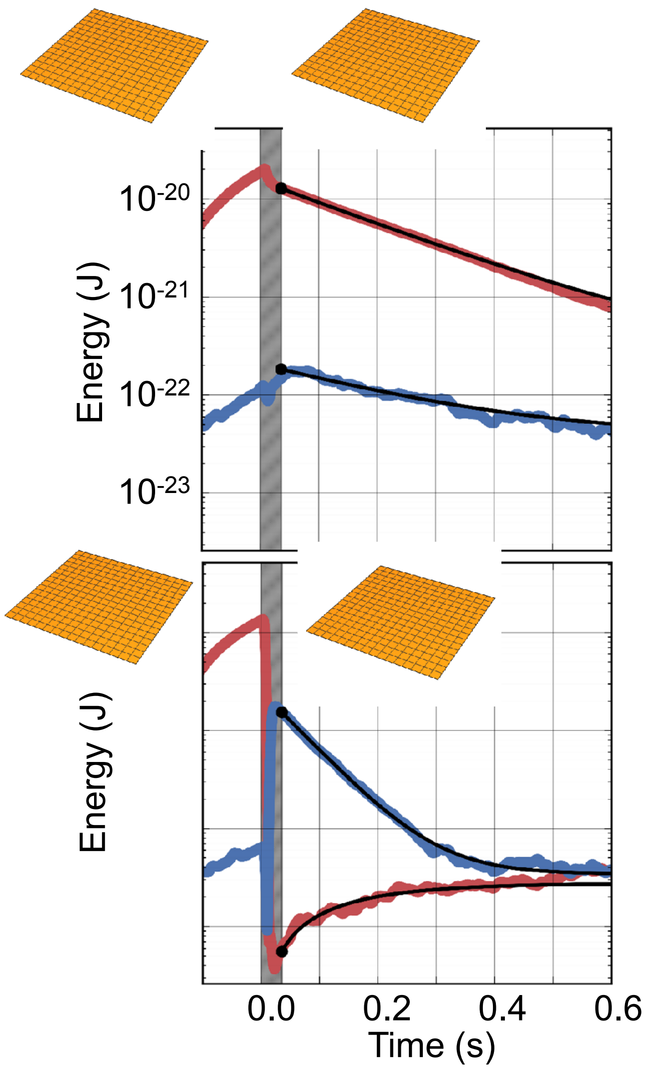
Topological Energy Transfer

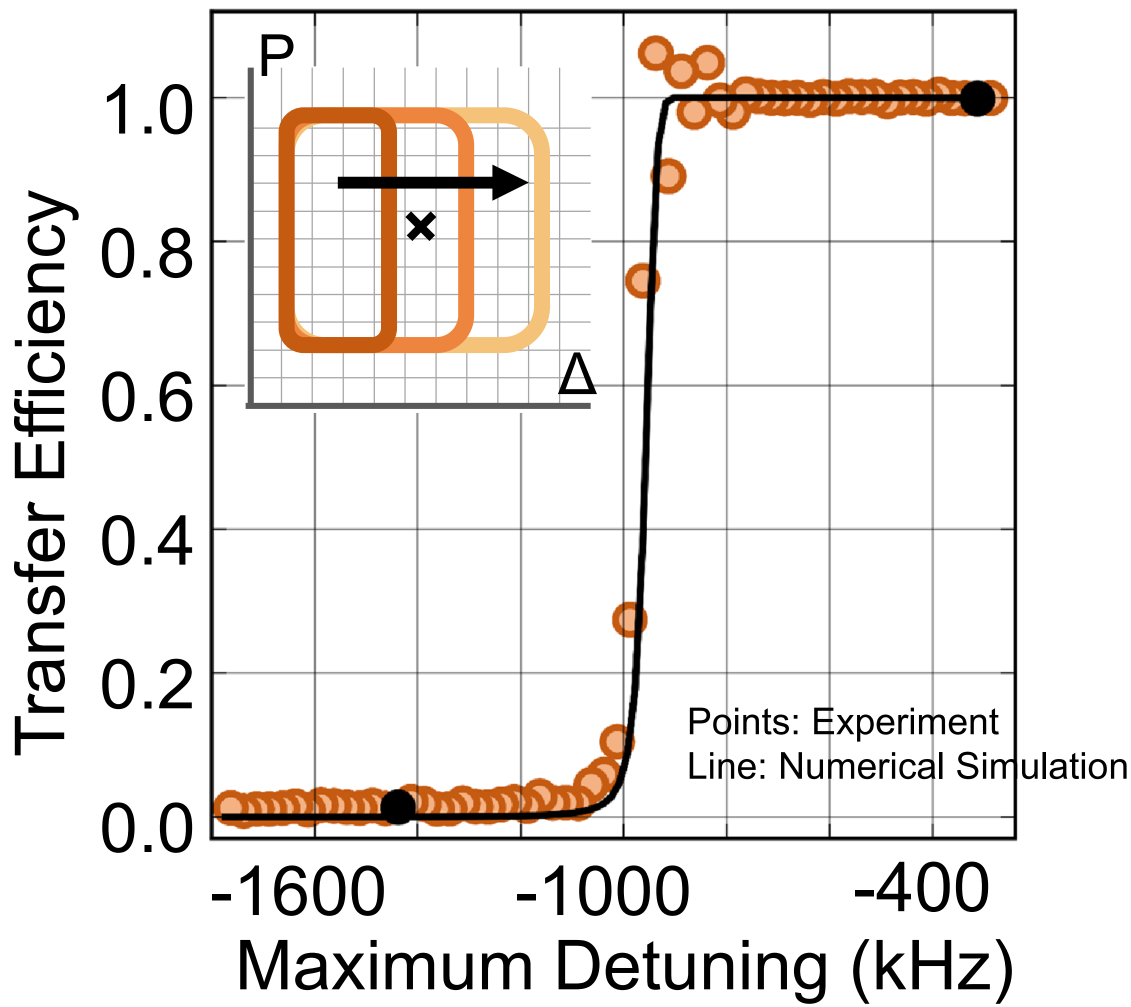




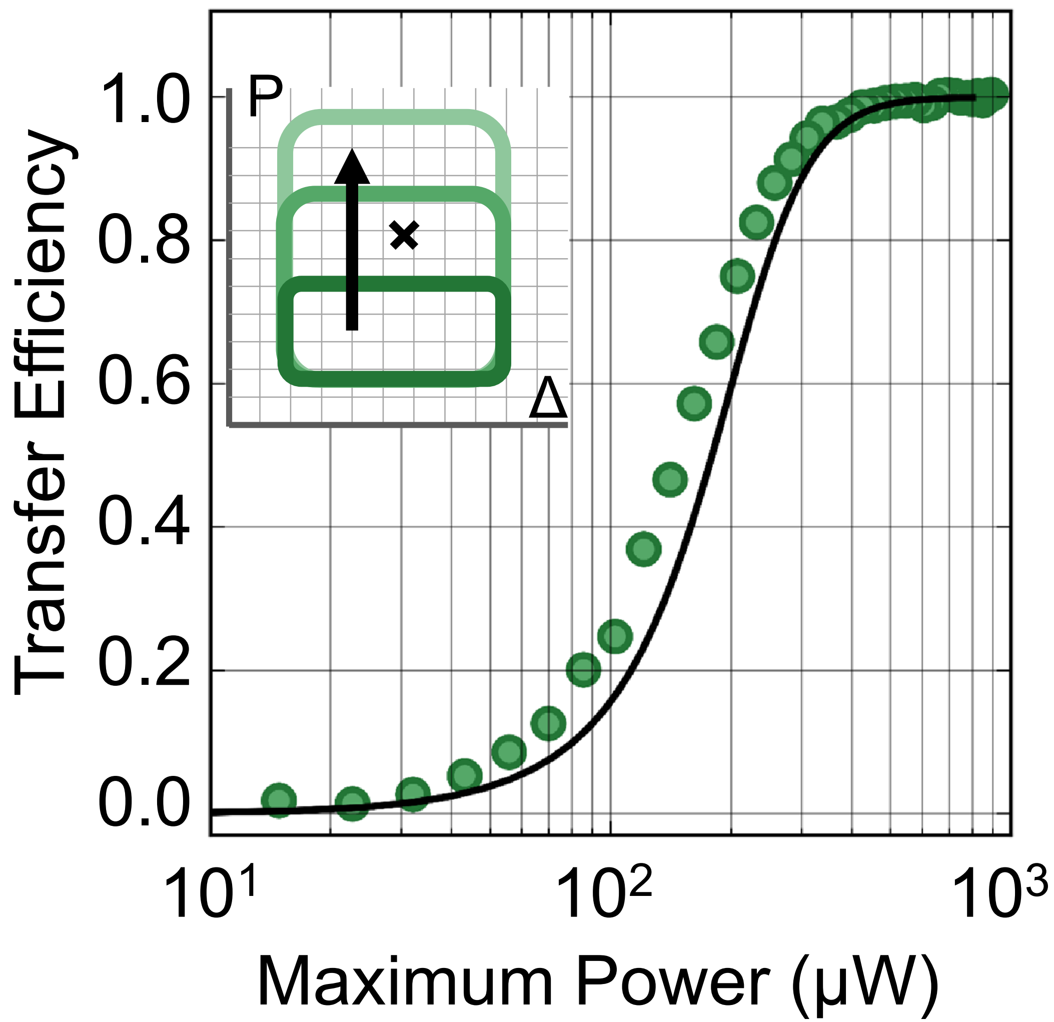
Dependence of the system’s adiabatic dynamics on the topology of the control loop

Topological Energy Transfer

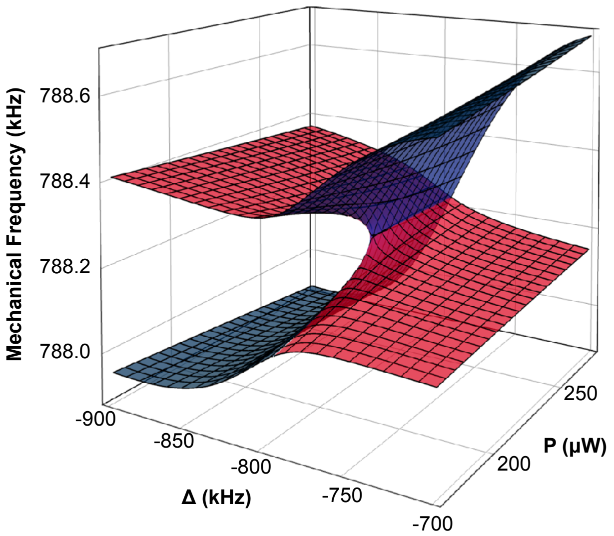


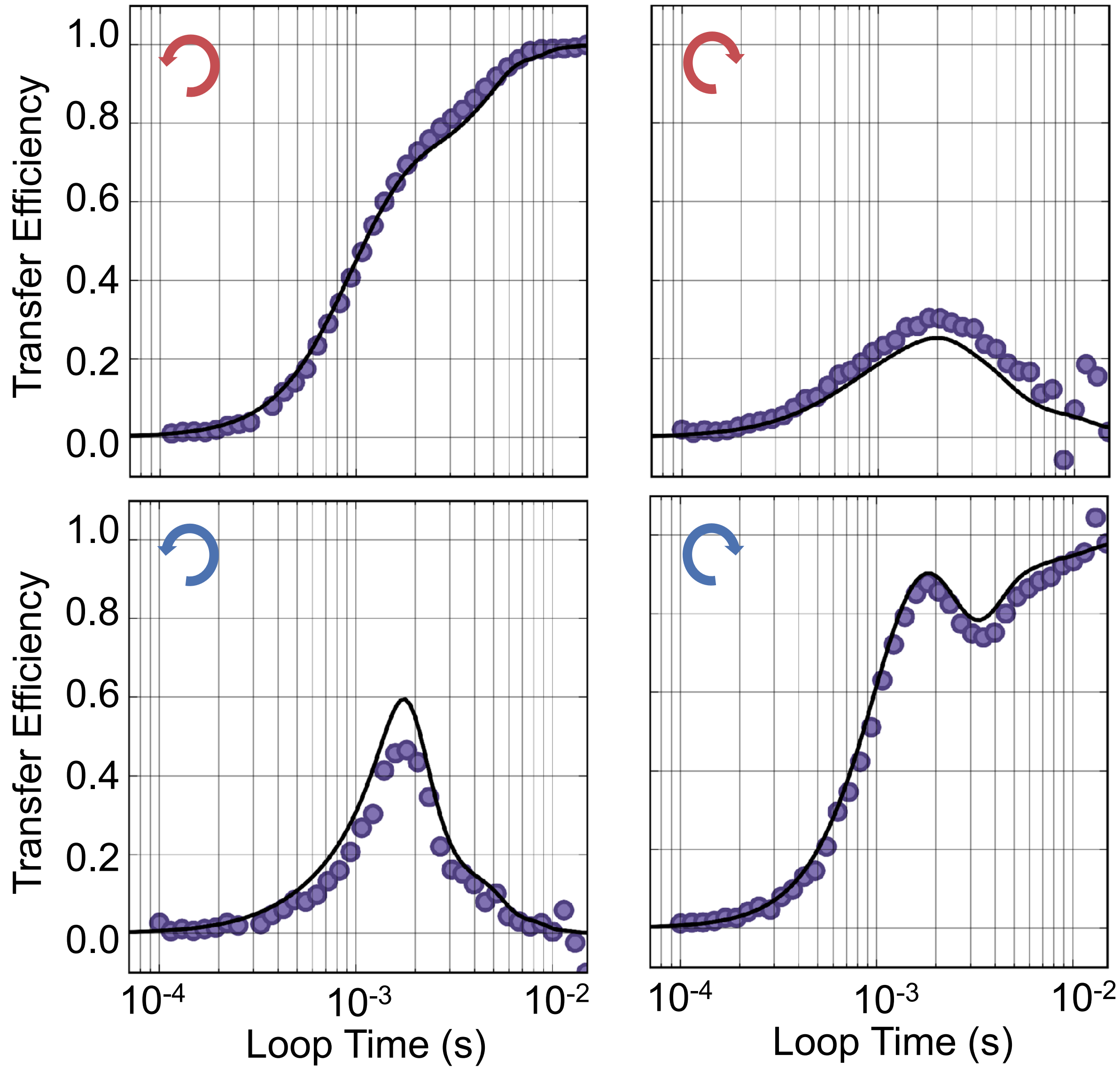


Topological Energy Transfer



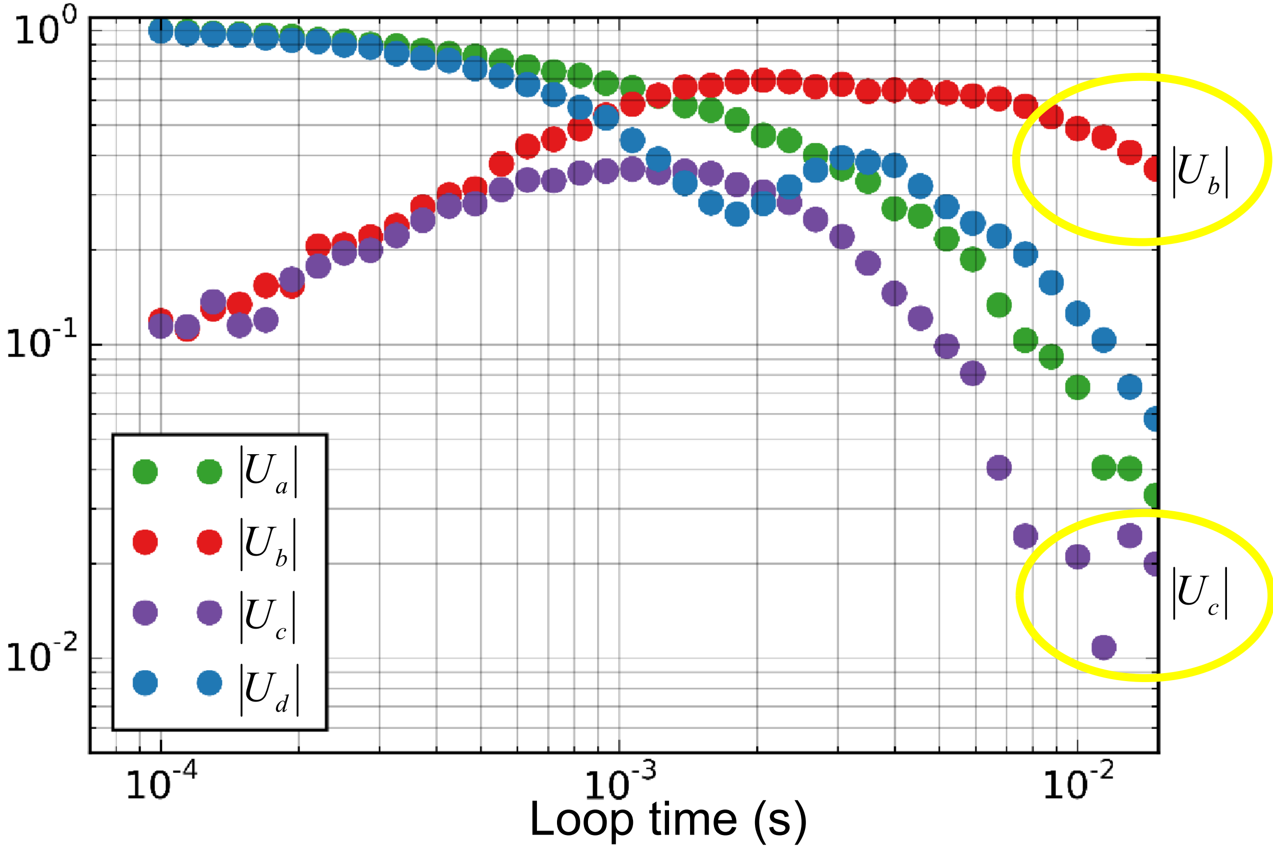
Non-reciprocal Topological Dynamics





Non-reciprocal Topological Dynamics

$$\hat{O}_{CW} = \begin{pmatrix} U_a & U_b \\ U_c & U_d \end{pmatrix}$$



$$|U_b| >> |U_c| \rightarrow \hat{O}_{CW} \neq \hat{O}_{CW}{}^T$$

Conclusions

- Observed an EP in an optomechanical system, and realized topological energy transfer by encircling the EP.
- Showed that the energy transfer depends on the topology of the encircling loop.
- Showed the transition from non-adiabatic to adiabatic energy transfer for increasing cycle time.
- Observed the breakdown of the usual adiabatic theorem and showed a diode-like asymmetry in the energy transfer.

Paper available on Nature, doi:10.1038/nature18604