Experiments testing macroscopic quantum spatial superpositions must be slow

(Scientific Reports (2016) – arXiv:1509.02408)

Andrea Mari, Giacomo De Palma, Vittorio Giovannetti

NEST - Scuola Normale Superiore and CNR-Nano, Pisa, Italy.


F=GMm
r2
M
m

Outline

Introduction

- Macroscopic superpositions
- Main result

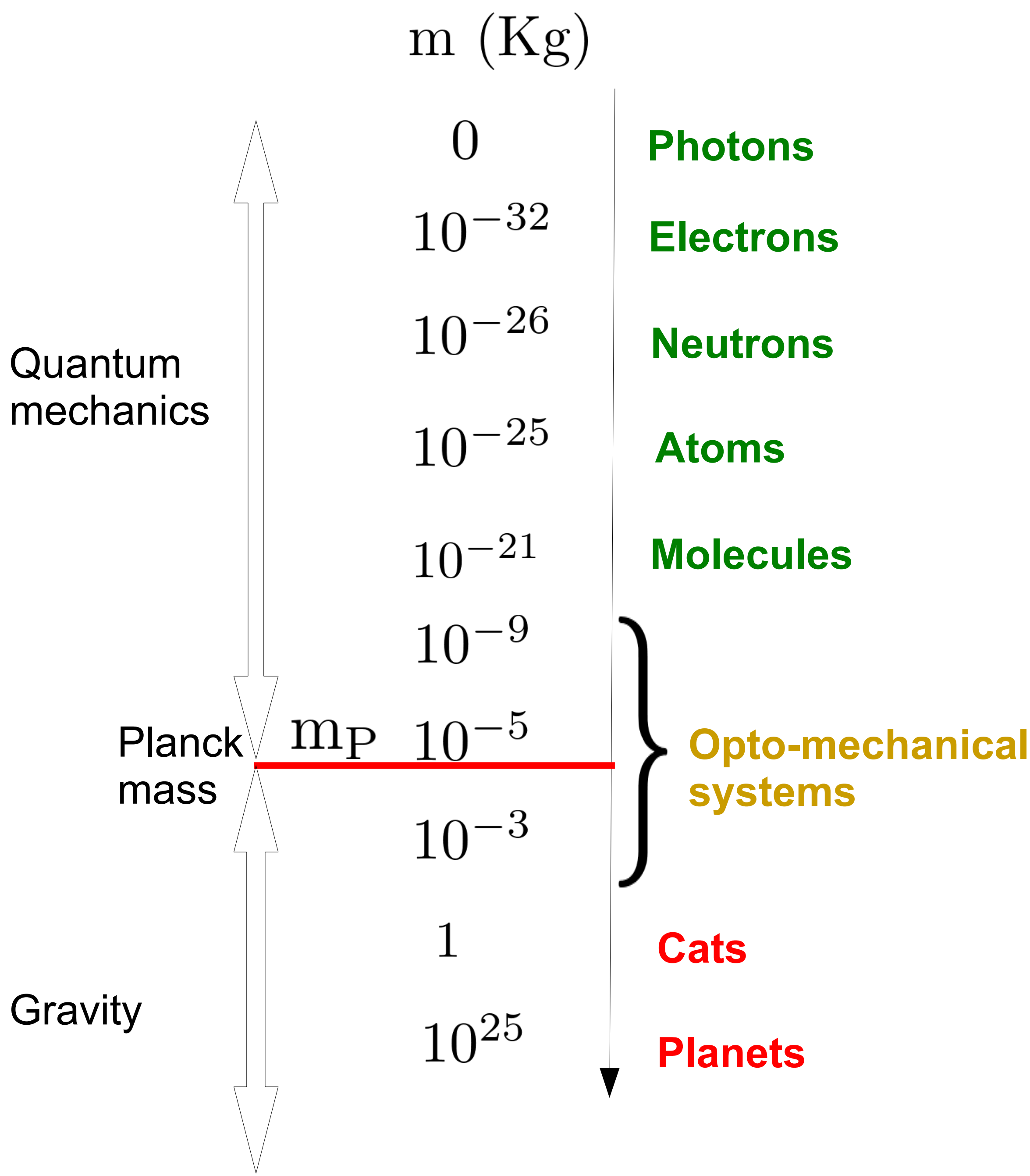
Derivation of the main result

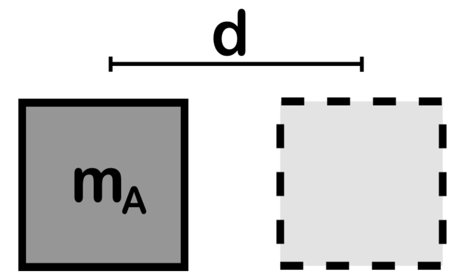
- A paradox
- Solution of the paradox
- Estimate of the minimum discrimination time

Implications

- Consistency with QED
- Implications for quantum gravity

Macroscopic spatial quantum superpositions





$$\psi(x) \propto \varphi(x) + \varphi(x-d)$$

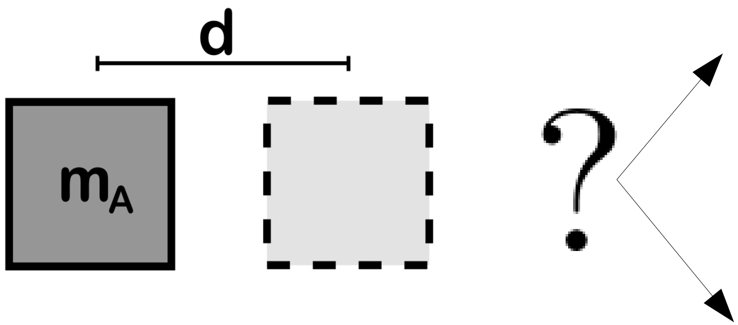
Legend

- **Experimentally observed**
- **Quantum effects observed but not yet superpositions**
- **Not observed**

$$m_P = \sqrt{\frac{\hbar c}{G}} \simeq 2.18 \times 10^{-8}\ \mathrm{kg}$$

The Problem: **Can we distinguish the quantum state from a classical one?**

Quantum spatial superposition of a mass m (or of a charge q)



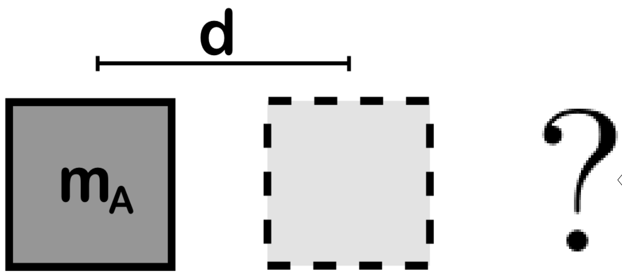
Quantum superposition $\rho^{(Q)} = |\psi\rangle\langle\psi|$

$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$

Classical mixture $\rho^{(C)} = \frac{1}{2}|L\rangle\langle L| + \frac{1}{2}|R\rangle\langle R|$

The Problem: **Can we distinguish the quantum state from a classical one?**

Quantum spatial superposition of a mass m (or of a charge q)



Quantum superposition $\rho^{(\mathrm{Q})} = |\psi\rangle\langle\psi|$

$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$

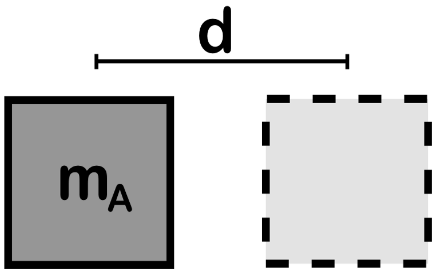
Classical mixture $\rho^{(\mathrm{C})} = \frac{1}{2}|L\rangle\langle L| + \frac{1}{2}|R\rangle\langle R|$

Quantum superposition of a macroscopic mass (or charge) — **consistency requirement** — No-signaling principle (relativistic causality)

The observation of macroscopic quantum superpositions requires a **minimum finite time**

Main result of this talk

Quantum spatial superposition of a mass m (or of a charge q)



$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$

Can we distinguish the quantum state from a classical one?

?

Quantum superposition $\rho^{(Q)} = |\psi\rangle\langle\psi|$

Classical mixture $\rho^{(C)} = \frac{1}{2}|L\rangle\langle L| + \frac{1}{2}|R\rangle\langle R|$

Result: *The minimum duration, of EVERY experiment, discriminating* $\rho^{(Q)}$ *from* $\rho^{(C)}$ *is:*

(superposition of a mass)

$$\mathrm{T} \gtrsim \frac{m}{m_{\mathrm{P}}} \frac{d}{c}$$

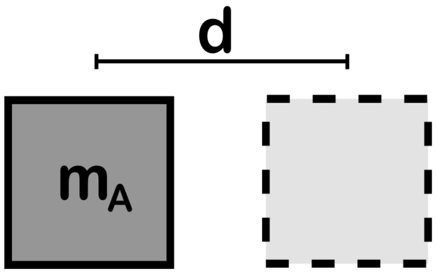
e.g.

m	d	T
1 g	1 mm	$1\ \mu\text{s}$
m_{earth}	$1\ \mu\text{m}$	age of the Universe!

$$\mathrm{m_P} = \sqrt{\frac{\hbar c}{G}} \simeq 2.18 \times 10^{-8} \text{ kg}$$

Main result of this talk

Quantum spatial superposition of a mass m (or of a charge q)



$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$

Can we distinguish the quantum state from a classical one?

?

Quantum superposition $\rho^{(Q)} = |\psi\rangle\langle\psi|$

Classical mixture $\rho^{(C)} = \frac{1}{2}|L\rangle\langle L| + \frac{1}{2}|R\rangle\langle R|$

Result: *The minimum duration, of EVERY experiment, discriminating* $\rho^{(Q)}$ *from* $\rho^{(C)}$ *is:*

(superposition of a mass)

$$\mathrm{T} \gtrsim \frac{m}{m_{\mathrm{P}}}\frac{d}{c}$$

e.g.

m	d	T
1 g	1 mm	1 μs
m_{earth}	1 μm	age of the Universe!

$$\mathrm{m_P} = \sqrt{\frac{\hbar c}{G}} \simeq 2.18 \times 10^{-8}\ \mathrm{kg}$$

(superposition of a charge)

$$\mathrm{T} \gtrsim \frac{q}{q_{\mathrm{P}}}\frac{d}{c}$$

e.g.

q	d	T
35 e	1 μm	10 fs
35 e	1 mm	10 ps

$$\mathrm{q_P} = \sqrt{4\pi\epsilon_0\hbar c} \simeq 11.7\ e \simeq 1.88 \times 10^{-18}\ \mathrm{C}$$

Outline

Introduction

- Macroscopic superpositions
- Main result

Derivation of the main result

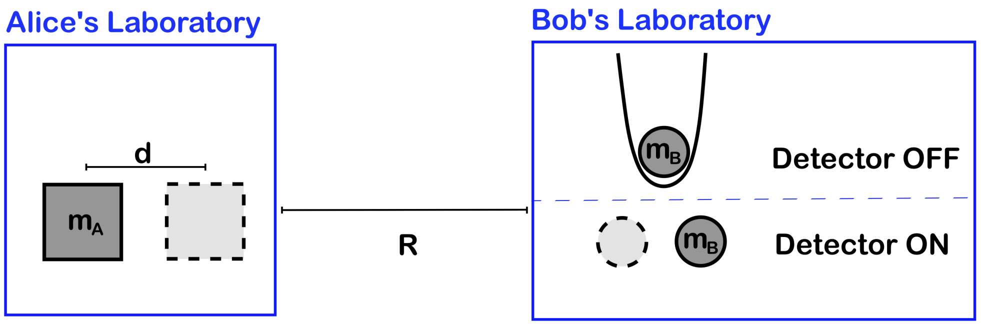
- A paradox
- Solution of the paradox
- Estimate of the minimum discrimination time

Implications

- Consistency with QED
- Implications for quantum gravity

A paradox

$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$



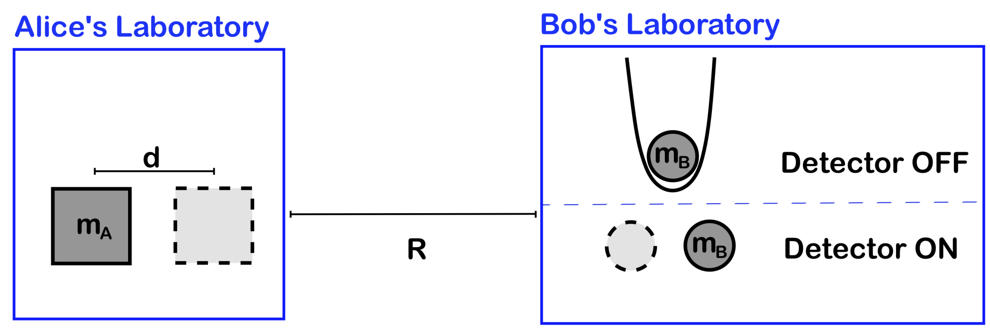
Protocol of the thought experiment

$t = -\infty$ Alice prepares a quantum macroscopic superposition;
Bob prepares a test mass in the ground state
of a very narrow harmonic trap

$$|\psi\rangle \otimes |\psi_B\rangle$$

A paradox

$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$



Protocol of the thought experiment

$t = -\infty$ Alice prepares a quantum macroscopic superposition; Bob prepares a test mass in the ground state of a very narrow harmonic trap

$$|\psi\rangle \otimes |\psi_B\rangle$$

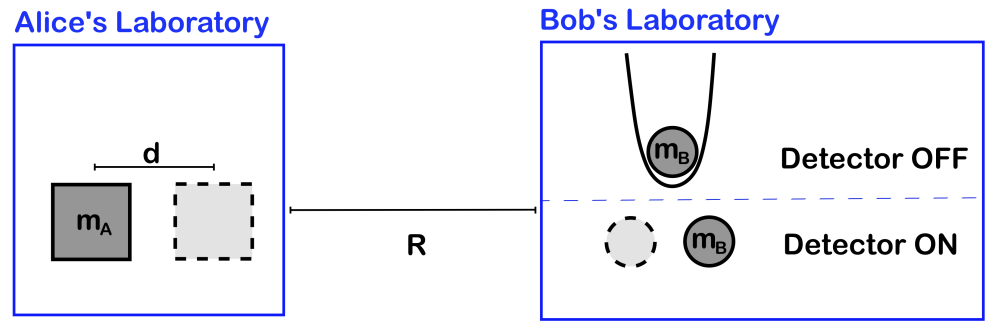
$t = 0$ Bob decides if :

- doing noting — No entanglement is created
- opening the trap — Entanglement creates after $t \geq T_B$

$$(|L\rangle|\ell\rangle + |R\rangle|r\rangle)/\sqrt{2}$$

A paradox

$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$



Protocol of the thought experiment

$t = -\infty$ Alice prepares a quantum macroscopic superposition; Bob prepares a test mass in the ground state of a very narrow harmonic trap

$$|\psi\rangle \otimes |\psi_B\rangle$$

$t = 0$ Bob decides if :

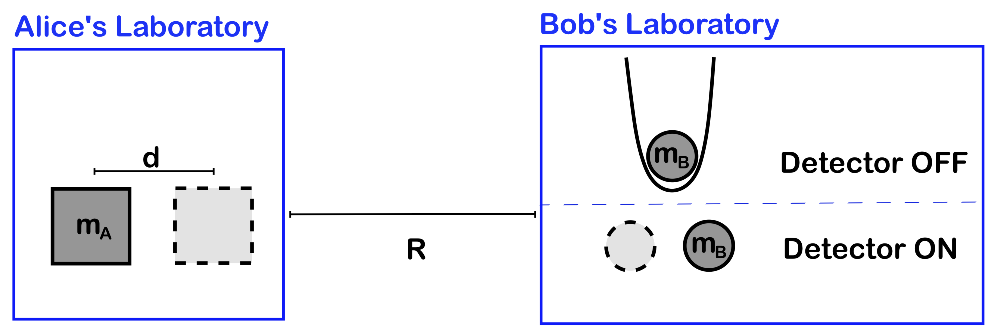
- doing noting — No entanglement is created
- opening the trap — Entanglement creates after $t \geq T_B$

$$(|L\rangle|\ell\rangle + |R\rangle|r\rangle)/\sqrt{2}$$

$t = T_{\rm B}$ Alice performs an arbitrary experiment aiming at discriminating $\rho^{(Q)}$ from $\rho^{(C)}$
In this way she deduces the choice of Bob.

A paradox

$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$



Protocol of the thought experiment

$t = -\infty$ Alice prepares a quantum macroscopic superposition.
Bob prepares a test mass in the ground state of a very narrow harmonic trap.

$t = 0$ Bob decides if :
- doing noting → No entanglement is created
- opening the trap → Entanglement creates after $t > T_{\rm B}$

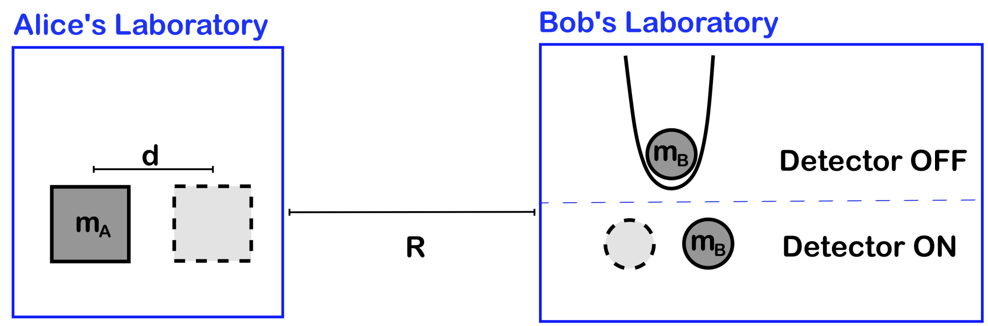
$t = T_{\rm B}$ Alice performs an arbitrary experiment aiming at discriminating $\rho^{(Q)}$ from $\rho^{(C)}$
In this way she deduces the choice of Bob.

Superluminal communication paradox

For sufficiently large $m_{\rm A}$ the entanglement generation time $T_{\rm B}$ can be arbitrarily reduced.

If $T_{\rm B} \leq \frac{R}{c}$ $\Longrightarrow$ Bob can send a signal to Alice faster than light !

Solution of the paradox



Alice can discriminate $\rho^{(Q)}$ from $\rho^{(C)}$, but the experiment must be slow!

Causality should be satisfied: $T_{\rm A} + T_{\rm B} \geq \frac{R}{c}$

Alice discrimination time

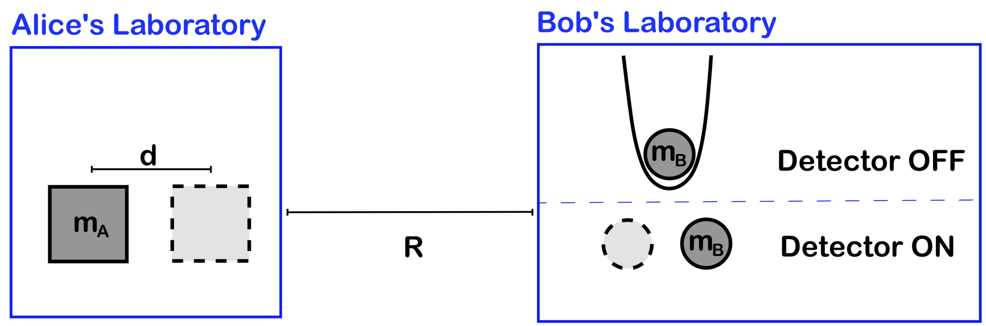
Bob measurement time

Free parameters of the thought experiment: (R , $m_{\rm B}$, width of the trap ΔX)

Let us choose them in order to get the best bound for $T_{\rm A}$

Estimate of the minimum discrimination time

$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$



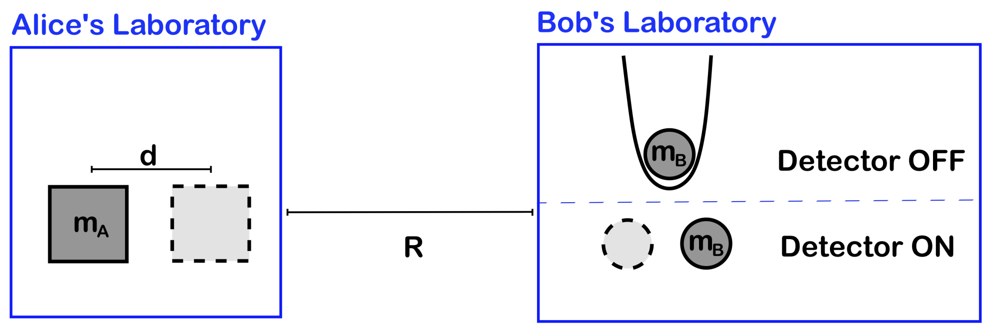
$$\Delta F = F_{\mathrm{L}} - F_{\mathrm{R}} \simeq \frac{G\, m_{\mathrm{A}}\, m_{\mathrm{B}}\, d}{R^3}$$

The two possible Hamiltonians for the free test mass in Bob's laboratory are:

$$\hat{H}_{\mathrm{L}} = \frac{\hat{P}^2}{2m_{\mathrm{B}}} - F_{\mathrm{L}}\hat{X} \ , \qquad \hat{H}_{\mathrm{R}} = \frac{\hat{P}^2}{2m_{\mathrm{B}}} - F_{\mathrm{R}}\hat{X}$$

Estimate of the minimum discrimination time

$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$



$$\Delta F = F_{\mathrm{L}} - F_{\mathrm{R}} \simeq \frac{G\, m_{\mathrm{A}}\, m_{\mathrm{B}}\, d}{R^3}$$

The two possible Hamiltonians for the free test mass in Bob's laboratory are:

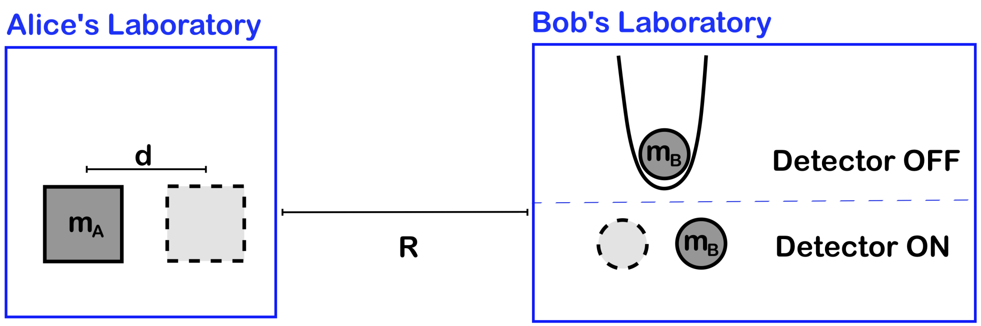
$$\hat{H}_{\mathrm{L}} = \frac{\hat{P}^2}{2m_{\mathrm{B}}} - F_{\mathrm{L}}\hat{X}\ , \qquad \hat{H}_{\mathrm{R}} = \frac{\hat{P}^2}{2m_{\mathrm{B}}} - F_{\mathrm{R}}\hat{X}$$

Entanglement is created when the two Hamiltonians drive the test mass into orthogonal states:

$$\left| \langle \psi_{\mathrm{B}} | e^{\frac{i}{\hbar}\hat{H}_{\mathrm{R}} t} e^{-\frac{i}{\hbar}\hat{H}_{\mathrm{L}} t} | \psi_{\mathrm{B}} \rangle \right| \ll 1$$

Estimate of the minimum discrimination time

$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$



$$\Delta F = F_{\mathrm{L}} - F_{\mathrm{R}} \simeq \frac{G\, m_{\mathrm{A}}\, m_{\mathrm{B}}\, d}{R^3}$$

The two possible Hamiltonians for the free test mass in Bob's laboratory are:

$$\hat{H}_{\mathrm{L}} = \frac{\hat{P}^2}{2m_{\mathrm{B}}} - F_{\mathrm{L}}\hat{X}\ , \qquad \hat{H}_{\mathrm{R}} = \frac{\hat{P}^2}{2m_{\mathrm{B}}} - F_{\mathrm{R}}\hat{X}$$

Entanglement is created when the two Hamiltonians drive the test mass into orthogonal states:

$$\left|\langle\psi_{\mathrm{B}}|\underbrace{e^{\frac{i}{\hbar}\hat{H}_{\mathrm{R}}t}e^{-\frac{i}{\hbar}\hat{H}_{\mathrm{L}}t}}|\psi_{\mathrm{B}}\rangle\right| \ll 1$$

(Baker–Campbell–Hausdorff formula)

$$\mathrm{e}^X e^Y = e^{X+Y+\frac{1}{2}[X,Y]+\frac{1}{12}([X,[X,Y]]-[Y,[X,Y]])+\ldots}$$

$$= \underbrace{\exp\left[i\,\frac{\Delta F}{\hbar}\left(\hat{X}\,t + \frac{\hat{P}}{2m_{\mathrm{B}}}t^2 + \cancel{\frac{F_1+F_2}{12m_{\mathrm{B}}}t^3}\right)\right]}$$

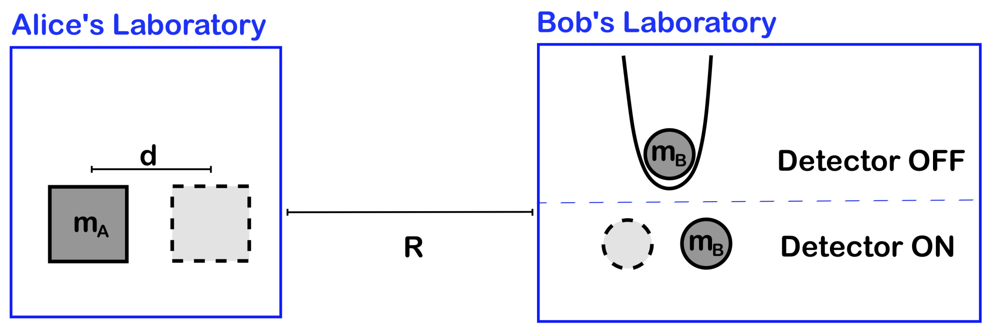
$$= e^{\frac{i}{\hbar}(\delta_x\hat{P} - \delta_p\hat{X})}$$ displacement operator

Displacement in position $\delta_x = \frac{\Delta F\, t^2}{2m_{\mathrm{B}}}$

Displacement in momentum $\delta_p = -\Delta F\, t$

Estimate of the minimum discrimination time (superposition of a mass)

$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$



$$\Delta F = F_{\mathrm{L}} - F_{\mathrm{R}} \simeq \frac{G\, m_{\mathrm{A}}\, m_{\mathrm{B}}\, d}{R^3}$$

The initial state of the test mass is Gaussian and characterized by $\begin{cases} \Delta X \simeq \text{ width of the trap} \\ \Delta P \simeq \dfrac{\hbar}{\Delta X} \end{cases}$

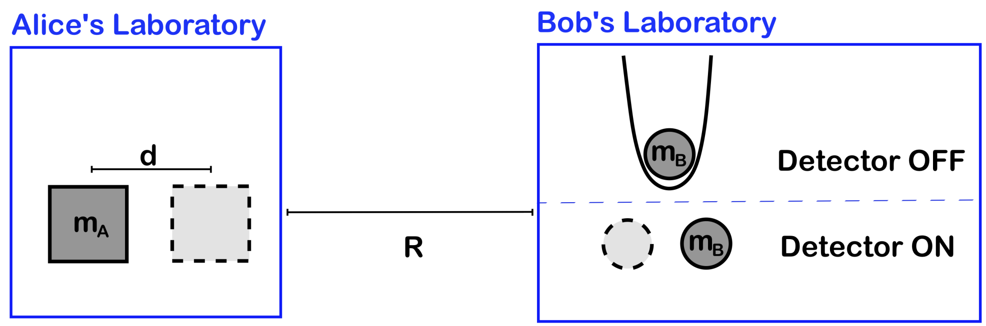
$$\left| \langle \psi_{\mathrm{B}} | e^{\frac{i}{\hbar}\hat{H}_{\mathrm{R}} t} e^{-\frac{i}{\hbar}\hat{H}_{\mathrm{L}} t} | \psi_{\mathrm{B}} \rangle \right| \ll 1$$

⇕

$\dfrac{\delta x}{\Delta X} \simeq 1$ or $\dfrac{\delta p}{\Delta P} \simeq 1$ where $\begin{cases} \delta_x = \dfrac{\Delta F\, t^2}{2 m_{\mathrm{B}}} \\ \delta_p = -\Delta F\, t \end{cases}$

Estimate of the minimum discrimination time (superposition of a mass)

$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$



$$\Delta F = F_{\mathrm{L}} - F_{\mathrm{R}} \simeq \frac{G\, m_{\mathrm{A}}\, m_{\mathrm{B}}\, d}{R^3}$$

The initial state of the test mass is Gaussian and characterized by $\begin{cases} \Delta X \simeq \text{ width of the trap} \\ \Delta P \simeq \dfrac{\hbar}{\Delta X} \end{cases}$

$$\left| \langle \psi_{\mathrm{B}} | e^{\frac{i}{\hbar}\hat{H}_{\mathrm{R}} t} e^{-\frac{i}{\hbar}\hat{H}_{\mathrm{L}} t} | \psi_{\mathrm{B}} \rangle \right| \ll 1$$

$$\frac{\delta x}{\Delta X} \simeq 1 \quad \text{or} \quad \frac{\delta p}{\Delta P} \simeq 1 \qquad \text{where} \quad \begin{cases} \delta_x = \dfrac{\Delta F\, t^2}{2 m_{\mathrm{B}}} \\ \delta_p = -\Delta F\, t \end{cases}$$

This condition is easier to get since the trap is very narrow.

$$\frac{\delta x}{\Delta X} = \frac{\Delta F\, T_{\mathrm{B}}^2}{2 m_{\mathrm{B}} \Delta X} \simeq 1$$

Maximum localization of a mass

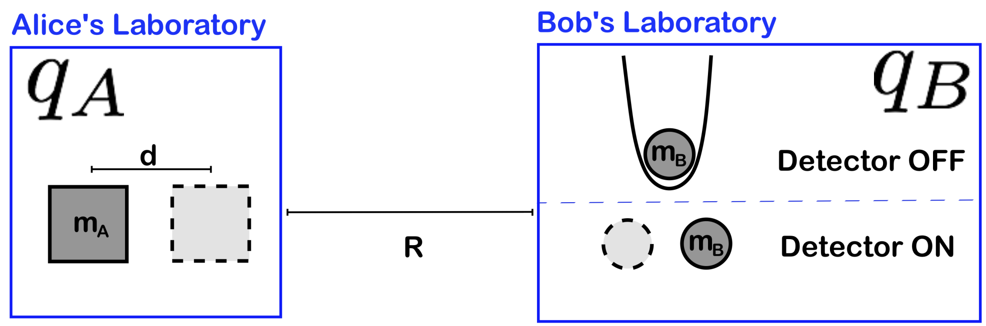
$$\Delta X \geq l_{\mathrm{P}} = \sqrt{\frac{\hbar G}{c^3}}$$

Causality inequality

$$T_{\mathrm{A}} + T_{\mathrm{B}} \geq \frac{R}{c} \implies T_{\mathrm{A}} \geq \frac{R}{c} - T_{\mathrm{B}} = \frac{2}{27} \frac{m_{\mathrm{A}}}{m_{\mathrm{P}}} \frac{d}{c}$$

Estimate of the minimum discrimination time (superposition of a charge)

$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$



$$\Delta F = F_{\rm L} - F_{\rm R} \simeq \frac{q_{\rm A}\, q_{\rm B}\, d}{4\pi\epsilon_0 R^3}$$

The initial state of the test mass is Gaussian and characterized by $\begin{cases} \Delta X \simeq \text{ width of the trap} \\ \Delta P \simeq \dfrac{\hbar}{\Delta X} \end{cases}$

$$\left|\langle\psi_{\rm B}|e^{\frac{i}{\hbar}\hat{H}_{\rm R} t} e^{-\frac{i}{\hbar}\hat{H}_{\rm L} t}|\psi_{\rm B}\rangle\right| \ll 1$$

$$\frac{\delta x}{\Delta X} \simeq 1 \quad \text{or} \quad \frac{\delta p}{\Delta P} \simeq 1 \qquad \text{where} \quad \begin{cases} \delta_x = \dfrac{\Delta F\, t^2}{2m_{\rm B}} \\ \delta_p = -\Delta F\, t \end{cases}$$

This condition is easier to get since the trap is very narrow.

$$\frac{\delta x}{\Delta X} = \frac{\Delta F\, T_{\rm B}^2}{2m_{\rm B}\Delta X} \simeq 1$$

Minimal radius of a charge

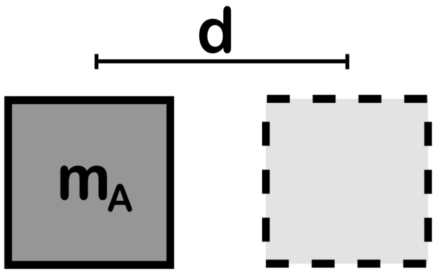
$$\Delta X \geq \frac{q_B}{q_{\rm P}} \frac{\hbar}{m_{\rm B} c}$$

Causality inequality

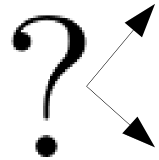
$$T_{\rm A} + T_{\rm B} \geq \frac{R}{c} \implies T_{\rm A} \geq \frac{R}{c} - T_{\rm B} = \frac{2}{27} \frac{q_{\rm A}}{q_{\rm P}} \frac{d}{c}$$

Estimate of the minimum discrimination time

Summary of the results



$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$



Quantum superposition $\rho^{(Q)} = |\psi\rangle\langle\psi|$

Classical mixture $\rho^{(C)} = \frac{1}{2}|L\rangle\langle L| + \frac{1}{2}|R\rangle\langle R|$

Result: *The minimum duration, of EVERY experiment, discriminating* $\rho^{(Q)}$ *from* $\rho^{(C)}$ *is:*

(superposition of a mass)

$$\mathrm{T} \gtrsim \frac{m}{m_{\mathrm{P}}} \frac{d}{c}$$

(superposition of a charge)

$$\mathrm{T} \gtrsim \frac{q}{q_{\mathrm{P}}} \frac{d}{c}$$

$$T \geq \max\{\frac{d}{c};\ \frac{m}{m_p}\frac{d}{c};\ \frac{q}{q_p}\frac{d}{c}\}$$

Outline

Introduction

- Macroscopic superpositions
- Main result

Derivation of the main result

- A paradox
- Solution of the paradox
- Estimate of the minimum discrimination time

Implications

- Consistency with QED
- Implications for quantum gravity

Consistency with QED

We have shown that

$$\mathrm{T_A} \gtrsim \frac{q}{q_\mathrm{P}} \frac{d}{c}$$

What is the physical origin of this bound?

Let us choose two specific experiments and see what happens.

How can we probe a spatial superposition ?

1) Interference experiment

2) Measure the momentum distribution

Consistency with QED

We have shown that $\mathrm{T_A} \gtrsim \frac{q}{q_\mathrm{P}} \frac{d}{c}$ **What is the physical origin of this bound?**

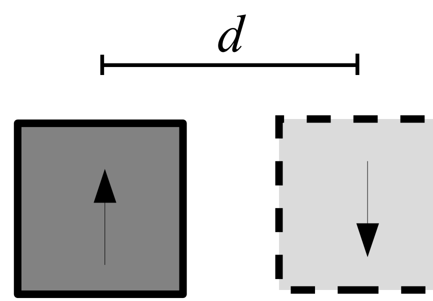
Let us choose two specific experiments and see what happens.

How can we probe a spatial superposition ? 1) Interference experiment

2) Measure the momentum distribution

1) Interference experiment

$$|\psi\rangle = \frac{|L\rangle|\uparrow\rangle + |R\rangle|\downarrow\rangle}{\sqrt{2}}$$



$t = 0$ Apply a spin dependent force which moves $|L\rangle$ to $|R\rangle$ within a time interval of $\mathrm{T_A}$

$$|\psi\rangle = |R\rangle \otimes \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$$

$t = T_\mathrm{A}$ Perform a spin measurement discriminating between

$$\rho_s^{(Q)} = \frac{|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow| + |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|}{2}$$

$$\rho_s^{(C)} = \frac{|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|}{2}$$

Consistency with QED

What happens if the experiment is too fast?

If the charge is accelerated **too much** it will radiate photons:

$$||\psi(0)\rangle = \frac{|L\rangle|\uparrow\rangle+|R\rangle|\downarrow\rangle}{\sqrt{2}} \otimes |0\rangle \longrightarrow ||\psi(0)\rangle = |R\rangle\frac{|\uparrow\rangle|\text{photons}\rangle+|\downarrow\rangle|0\rangle}{\sqrt{2}}$$

vacuum radiation field

$$\rho_s^{(C)} = \frac{|\uparrow\rangle\langle\uparrow|+|\downarrow\rangle\langle\downarrow|}{2}$$

Consistency with QED

What happens if the experiment is too fast?

If the charge is accelerated **too much** it will radiate photons:

$$||\psi(0)\rangle = \frac{|L\rangle|\uparrow\rangle + |R\rangle|\downarrow\rangle}{\sqrt{2}} \otimes |0\rangle \longrightarrow ||\psi(0)\rangle = |R\rangle \frac{|\uparrow\rangle|\text{photons}\rangle + |\downarrow\rangle|0\rangle}{\sqrt{2}}$$

vacuum radiation field

$$\rho_s^{(C)} = \frac{|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|}{2}$$

What is the minimum time such that radiation is not produced?

Consistency with QED

What happens if the experiment is too fast?

If the charge is accelerated **too much** it will radiate photons:

$$\|\psi(0)\rangle = \frac{|L\rangle|\uparrow\rangle + |R\rangle|\downarrow\rangle}{\sqrt{2}} \otimes |0\rangle \longrightarrow \|\psi(0)\rangle = |R\rangle \frac{|\uparrow\rangle|\text{photons}\rangle + |\downarrow\rangle|0\rangle}{\sqrt{2}}$$

vacuum radiation field

$$\rho_s^{(C)} = \frac{|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|}{2}$$

What is the minimum time such that radiation is not produced?

Non-trivial QED calculation $\longrightarrow$

$$\boxed{\mathrm{T_A} \simeq \frac{q}{q_{\mathrm{P}}}\,\frac{d}{c}}$$

Implications for quantum gravity

For superpositions of **charged systems** we have just shown:

$$\mathrm{T_A} \gtrsim \frac{q}{q_\mathrm{P}} \frac{d}{c}$$

What is the physical origin of this bound?

1) Photons
2) Vacuum fluctuations

For superpositions of **massive systems**, the analogy with QED would suggest:

$$\mathrm{T}_A \gtrsim \frac{m}{m_\mathrm{P}} \frac{d}{c}$$

What is the physical origin of this bound?

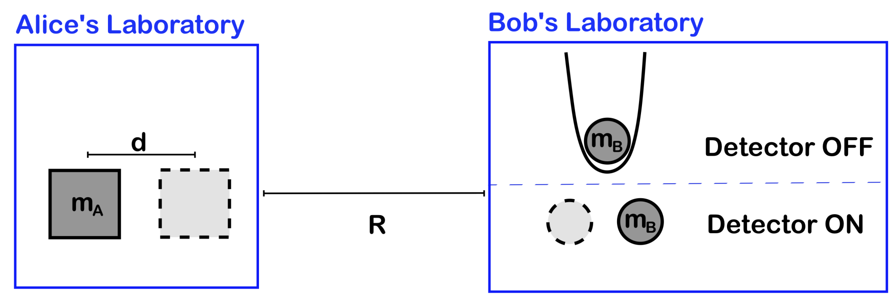
1) Gravitons !
2) Metric fluctuations !

Conclusions

"No progress without a paradox"

Experiments testing macroscopic quantum superpositions must be slow:

$$T_A \gtrsim \frac{q}{q_P}\frac{d}{c} \qquad T_A \gtrsim \frac{m}{m_P}\frac{d}{c}$$



- Fully consistent with quantum electrodynamics
- Indirect evidence of a quantum gravity effects: gravitons, metric fluctuations.
- Above a certain scale macroscopic superpositions are not observable

Outlook

- Use linearized quantum gravity to verify the bound
- Other *thought experiments ?*

Thanks!!!

Mari, De Palma, Giovannetti, *Sci. Rep. 6, 22777 (2016)*

Supplementary Slides

Consistency with QED

2) Measure the momentum distribution (second experiment)

$$\psi(\mathbf{x}) = \frac{\phi(\mathbf{x}) + e^{i\varphi}\,\phi(\mathbf{x}-\mathbf{d})}{\sqrt{2}} \xrightarrow{\text{momentum distrib.}} \frac{|\psi(\mathbf{k})|^2}{(2\pi)^3} = 2\cos^2\Big(\underbrace{\frac{\mathbf{k}\cdot\mathbf{d}-\varphi}{2}}_{\pi}\Big)\,\frac{|\phi(\mathbf{k})|^2}{(2\pi)^3}$$

Interference fringes with distance of the order of $\frac{\pi}{d}$

The precision required in the measurement of momentum is $\boxed{\Delta P \lesssim \frac{\pi\hbar}{d}}$

Consistency with QED

2) Measure the momentum distribution (second experiment)

$$\psi(\mathbf{x}) = \frac{\phi(\mathbf{x}) + e^{i\varphi}\,\phi(\mathbf{x}-\mathbf{d})}{\sqrt{2}} \xrightarrow{\text{momentum distrib.}} \frac{|\psi(\mathbf{k})|^2}{(2\pi)^3} = 2\cos^2\Big(\underbrace{\frac{\mathbf{k}\cdot\mathbf{d}-\varphi}{2}}_{\pi}\Big)\,\frac{|\phi(\mathbf{k})|^2}{(2\pi)^3}$$

Interference fringes with distance of the order of $\dfrac{\pi}{d}$

The precision required in the measurement of momentum is $\boxed{\Delta P \lesssim \dfrac{\pi\hbar}{d}}$

From the minimal coupling Hamiltonian $\hat{\mathbf{P}} = m\,\hat{\mathbf{V}} + q\,\hat{\mathbf{A}}\left(\hat{\mathbf{X}}\right)$

The velocity is gauge invariant and locally measurable

"Noise" term with infinite variance !

$$\langle 0|\hat{\mathbf{A}}\left(\hat{\mathbf{X}}\right)^2|0\rangle = \left(\int \frac{1}{|\mathbf{k}|}\,\frac{d^3k}{(2\pi)^3}\right)\hat{1}_{\mathrm{A}}$$

Consistency with QED

2) Measure the momentum distribution (second experiment)

$$\psi(\mathbf{x}) = \frac{\phi(\mathbf{x}) + e^{i\varphi}\,\phi(\mathbf{x}-\mathbf{d})}{\sqrt{2}} \xrightarrow{\text{momentum distrib.}} \frac{|\psi(\mathbf{k})|^2}{(2\pi)^3} = 2\cos^2\Big(\underbrace{\frac{\mathbf{k}\cdot\mathbf{d}-\varphi}{2}}_{\pi}\Big)\,\frac{|\phi(\mathbf{k})|^2}{(2\pi)^3}$$

Interference fringes with distance of the order of $\dfrac{\pi}{d}$

The precision required in the measurement of momentum is $\boxed{\Delta P \lesssim \dfrac{\pi\hbar}{d}}$

From the minimal coupling Hamiltonian $\hat{\mathbf{P}} = m\,\hat{\mathbf{V}} + q\,\hat{\mathbf{A}}\left(\hat{\mathbf{X}}\right)$

The velocity is gauge invariant and locally measurable

"Noise" term with infinite variance !

$$\langle 0|\hat{\mathbf{A}}\left(\hat{\mathbf{X}}\right)^2|0\rangle = \left(\int \frac{1}{|\mathbf{k}|}\,\frac{d^3k}{(2\pi)^3}\right)\hat{1}_{\mathrm{A}}$$

Slow measurement of averaged velocity

Averaged noise:

$$\hat{\mathbf{A}}_{av} = \int \hat{\mathbf{A}}\left(\hat{\mathbf{X}}, t\right)\,\varphi(t)\,dt \qquad \varphi(t) = \frac{e^{-\frac{t^2}{2T^2}}}{\sqrt{2\pi}\,T}$$

$$\boxed{\langle 0|\hat{\mathbf{A}}_{av}^2|0\rangle = \frac{\hat{1}_{\mathrm{A}}}{4\pi^2T^2}} \Longrightarrow T \gtrsim \frac{1}{\sqrt{3\pi^3}}\,\frac{q}{q_{\mathrm{P}}}\,\frac{d}{c}$$

The same bound, again!

Planck units

In this talk:

Planck mass: $\mathrm{m_P} = \sqrt{\frac{\hbar c}{G}} \simeq 2.18 \times 10^{-8}\ \mathrm{kg}$

Planck length: $\mathrm{l_P} = \sqrt{\frac{\hbar G}{c^3}} \simeq 1.6 \times 10^{-35}\mathrm{m}$

Planck charge: $\mathrm{q_P} = \sqrt{4\pi\epsilon_0 \hbar c} \simeq 11.7\ e \simeq 1.88 \times 10^{-18}\ \mathrm{C}$ (~ 12 positrons)

Physical operational interpretations:

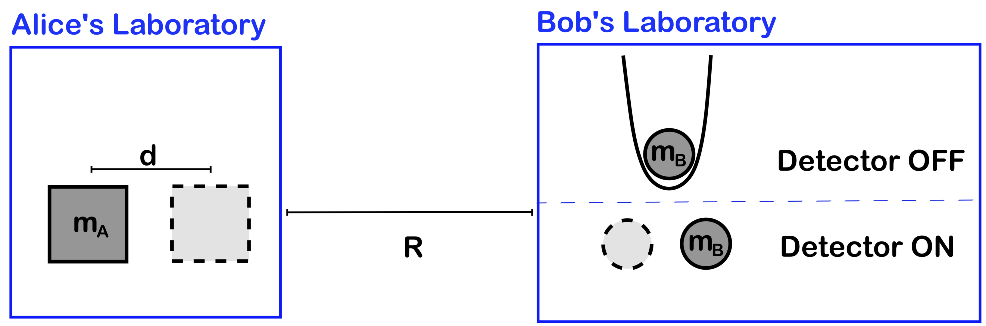
Quantum gravity is relevant for: $m \geq m_{\mathrm{P}}$

Minimal universal length: $\Delta X \geq l_{\mathrm{P}}$

Minimal radius for a charge: $\Delta X \geq \frac{q}{q_{\mathrm{P}}} \frac{\hbar}{mc}$

Estimate of the minimum discrimination time (superposition of a mass)

$$|\psi\rangle = \frac{|L\rangle + |R\rangle}{\sqrt{2}}$$



$$\Delta F = F_{\mathrm{L}} - F_{\mathrm{R}} \simeq \frac{G\, m_{\mathrm{A}}\, m_{\mathrm{B}}\, d}{R^3}$$

The initial state of the test mass is Gaussian and characterized by $\begin{cases} \Delta X \simeq \text{ width of the trap} \\ \Delta P \simeq \dfrac{\hbar}{\Delta X} \end{cases}$

$$\left| \langle \psi_{\mathrm{B}} | e^{\frac{i}{\hbar}\hat{H}_{\mathrm{R}} t} e^{-\frac{i}{\hbar}\hat{H}_{\mathrm{L}} t} | \psi_{\mathrm{B}} \rangle \right| \ll 1$$

$$\Updownarrow$$

$$\frac{\delta x}{\Delta X} \simeq 1 \quad \text{or} \quad \frac{\delta p}{\Delta P} \simeq 1 \qquad \text{where} \quad \begin{cases} \delta_x = \dfrac{\Delta F\, t^2}{2 m_{\mathrm{B}}} \\ \delta_p = -\Delta F\, t \end{cases}$$

This condition is easier to get since the trap is very narrow.

$$\Updownarrow$$

$$\frac{\delta x}{\Delta X} = \frac{\Delta F\, T_{\mathrm{B}}^2}{2 m_{\mathrm{B}} \Delta X} \simeq 1$$

Maximum localization of a mass

$$\Delta X \geq l_{\mathrm{P}} = \sqrt{\frac{\hbar G}{c^3}}$$

$$\Updownarrow$$

$$0 \leq \eta \equiv \frac{c\, T_{\mathrm{B}}}{R} \leq 1 \qquad T_{\mathrm{B}} \simeq \frac{1}{2}\eta^3 \frac{m_{\mathrm{A}}}{m_{\mathrm{P}}} \frac{d}{c}$$

Causality inequality $T_{\mathrm{A}} + T_{\mathrm{B}} \geq \dfrac{R}{c} \implies T_{\mathrm{A}} \geq \dfrac{T_{\mathrm{B}}}{\eta} - T_{\mathrm{B}} = \dfrac{1}{2} \dfrac{m_{\mathrm{A}}}{m_{\mathrm{P}}} \dfrac{d}{c} (\eta^2 - \eta^3)\Big|_{\eta = \frac{2}{3}} = \dfrac{2}{27} \dfrac{m_{\mathrm{A}}}{m_{\mathrm{P}}} \dfrac{d}{c}$

Consistency with QED

What happens if the experiment is too fast?

If the charge is accelerated **too much** it will radiate photons:

$$\|\psi(0)\rangle = \frac{|L\rangle|\uparrow\rangle + |R\rangle|\downarrow\rangle}{\sqrt{2}} \otimes |0\rangle \longrightarrow \|\psi(0)\rangle = |R\rangle \frac{|\uparrow\rangle|\text{photons}\rangle + |\downarrow\rangle|0\rangle}{\sqrt{2}}$$

vacuum radiation field

$$\rho_s^{(C)} = \frac{|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|}{2}$$

What is the minimum time such that radiation is not produced?

Non-trivial QED calculation $\longrightarrow$ $\mathrm{T_A} \simeq \frac{q}{q_\mathrm{P}} \frac{d}{c}$ **bound saturated !**

Sketch of the calculation:

Fix the trajectory of the charge to be *e.g.* $\mathrm{x(t){=}d}\ \sin^2\left(\frac{\pi}{2}\frac{t}{t_0}\right)$ for $0 \leq t \leq t_0$

Classical current density $\mathrm{J(k,t)} \simeq q\ \mathbf{v}(t) \longrightarrow |0\rangle \to |f\rangle$ (coherent field)

$$|\langle 0|f\rangle|^2 = \exp\left(-\frac{q^2}{6\pi^2}\int_0^\infty |\mathbf{v}(\omega)|^2\ \omega\ d\omega\right) \simeq \exp\left(-2\ \frac{q^2}{q_\mathrm{P}^2}\ \frac{d^2}{c^2\ t_0^2}\right)$$