Breaking reciprocity through optomechanical interactions

Freek Ruesink

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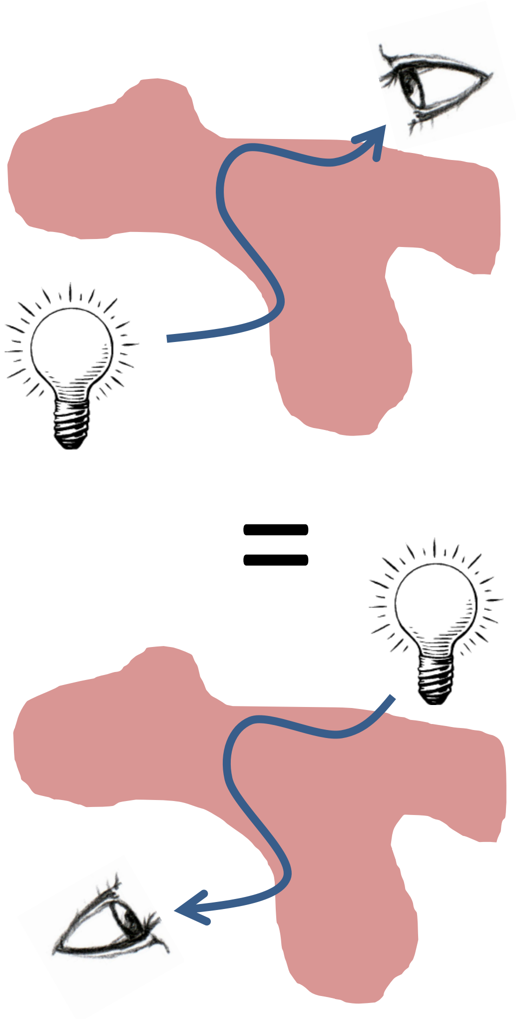
Lorentz reciprocity in optics

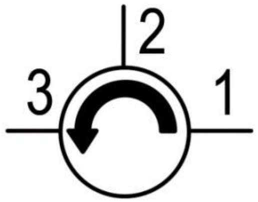


- Reciprocity: transmission through system is equal when source and detector are interchanged

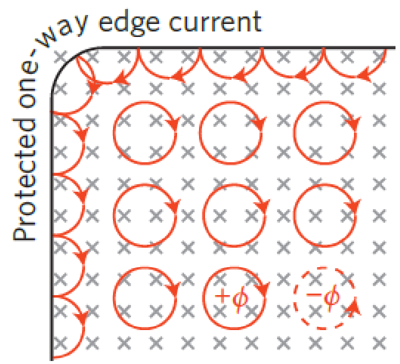
- Rooted in:
  - Linearity
  - Time-reversal symmetry

Lorentz reciprocity in optics

- **Nonreciprocal elements**: Isolators, nonreciprocal phase shifters, circulators



- **Nonreciprocal materials**: quantum Hall effect, topological insulator



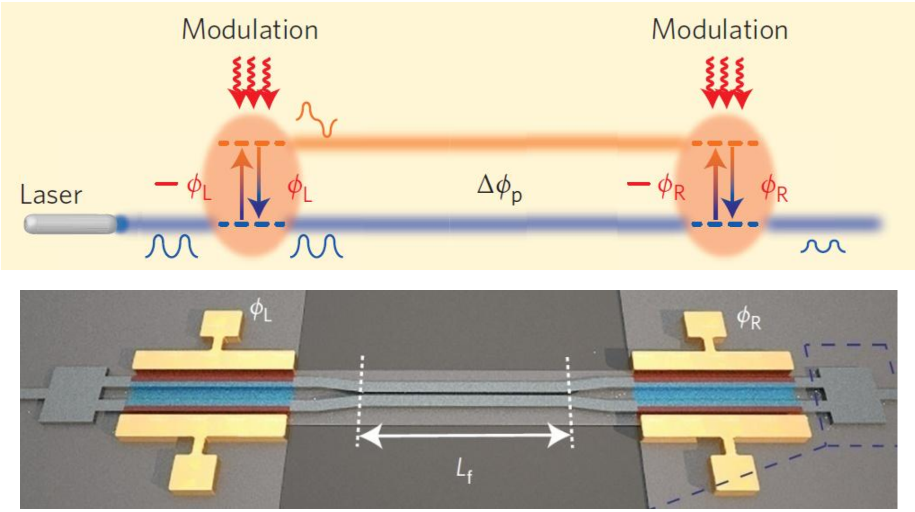
Lu, Joannopoulos, Soljacic, Topological photonics, Nature Photon. 8, 821 (2014)

Jalas, D. *et al.* What is — and what is not — an optical isolator. *Nat. Photonics* **7,** 579–582 (2013).

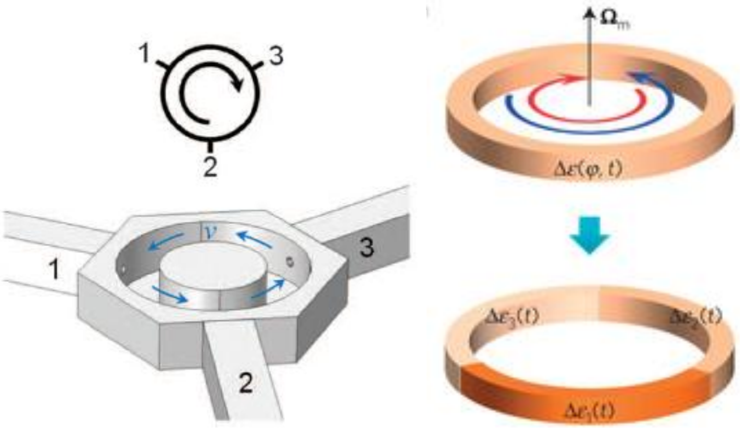
Breaking reciprocity

Breaking time-reversal symmetry through temporal modulation

Fang, Yu, Fan, Phys. Rev. Lett. 108, 153901 (2012), Nature Photon. 6, 782 (2012)



Lipson, Fan, Nature Photon. 2014



Alù, Science 2014, Nature Phys. 2014

Breaking reciprocity

Breaking time-reversal symmetry through temporal modulation

… using optomechanical interactions?

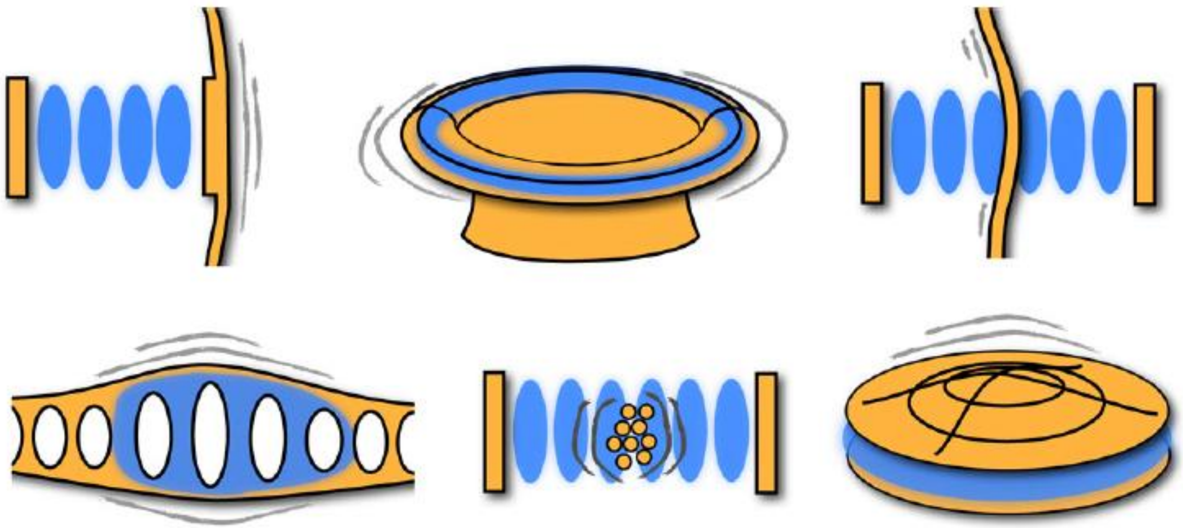


image: Aspelmeyer et al., Rev. Mod. Phys. 86, 1391 (2014)

Breaking reciprocity

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Mode conversion:

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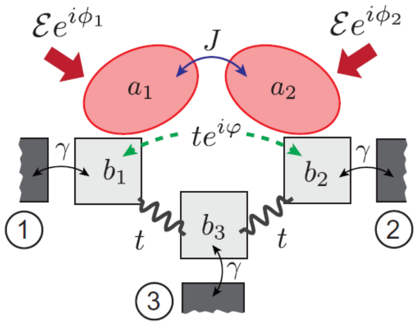
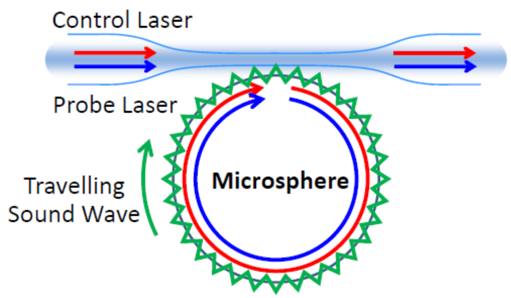
Schmidt et al., Optica 2, 635 (2015)



Ring resonators:

Hafezi & Rabl, Opt. Express 20, 7672 (2012)
Kim et al., Nature Phys. 11, 275 (2015)
Shen et al., arXiv:1604.02297 (2016)



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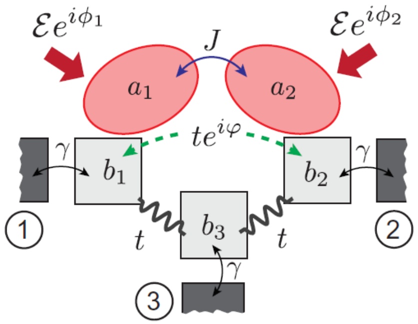

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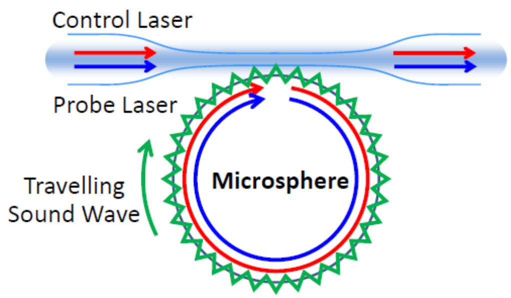

This work:

- Minimal requirements for optomechanical nonreciprocity
- Quantitative nonreciprocal transmittance measurements

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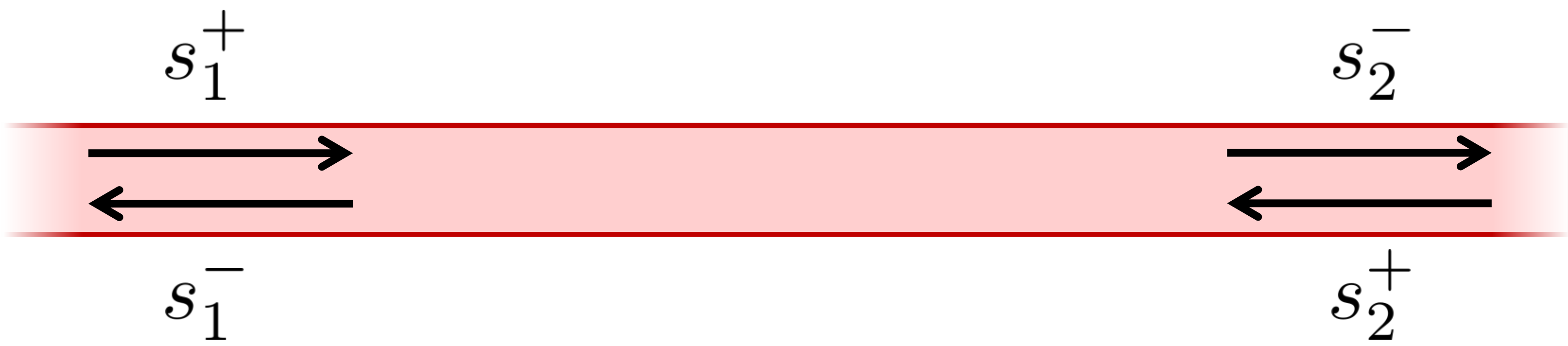
Nonreciprocity and the S matrix

Nonreciprocity requires an asymmetric scattering matrix S

$$\begin{pmatrix} s_1^- \\ s_2^- \end{pmatrix} = S \begin{pmatrix} s_1^+ \\ s_2^+ \end{pmatrix} \qquad S = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}$$

$$s_{12} = s_{21}$$



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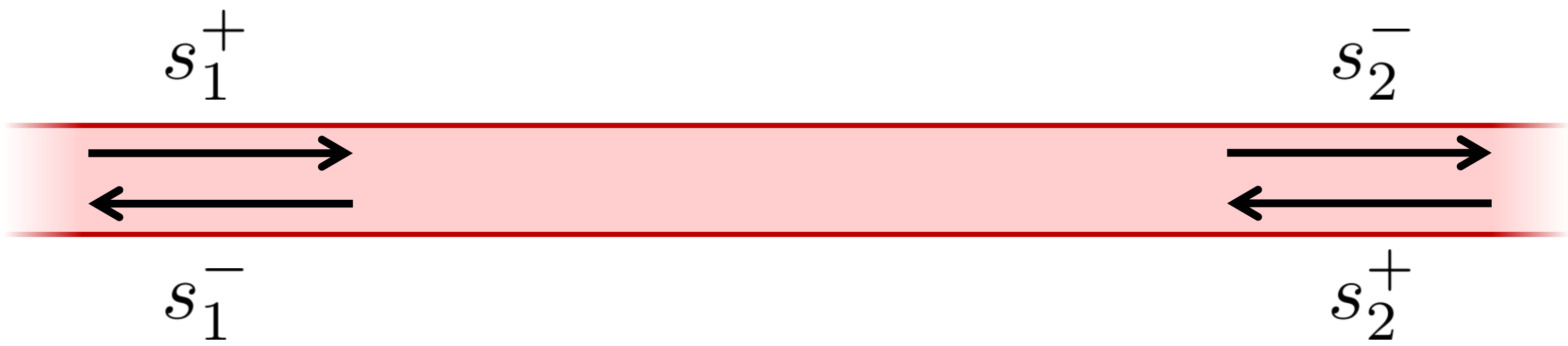
$$s_{12} \neq s_{21}$$

Create asymmetry



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Nonreciprocity in a two mode system

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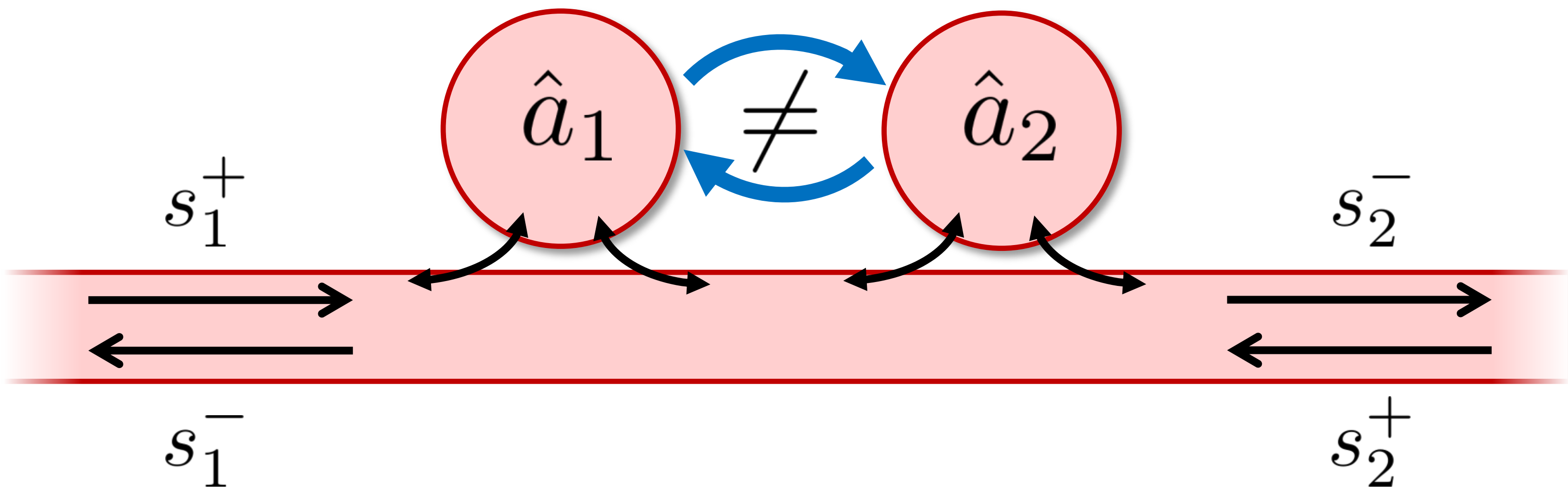
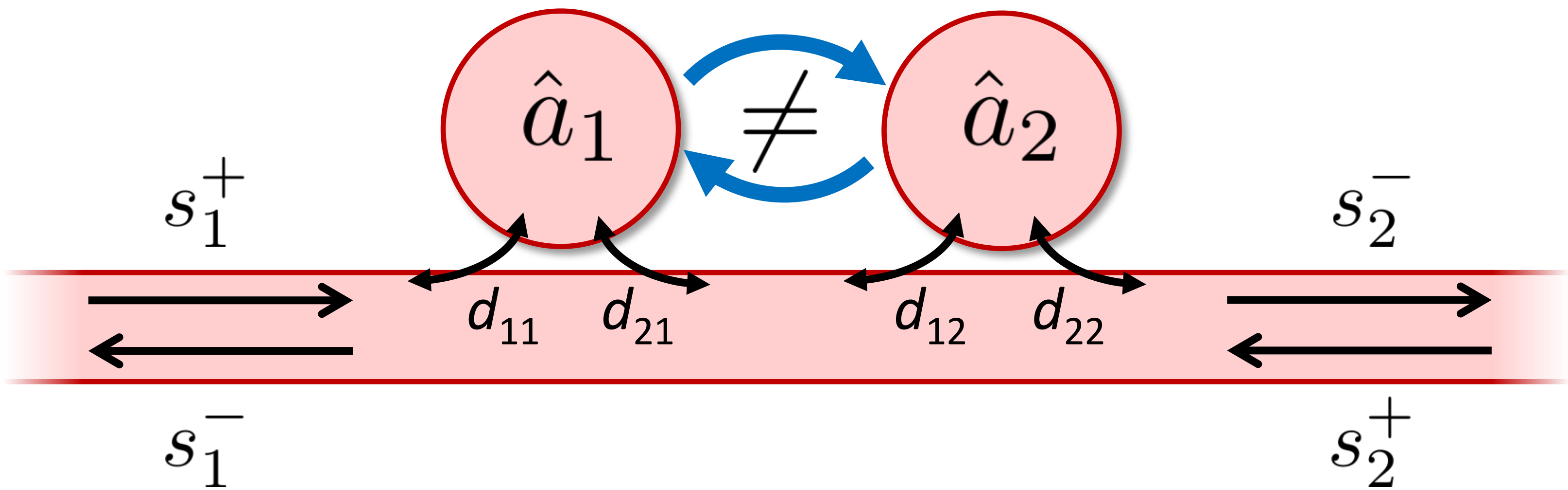


Nonreciprocity in a two mode system

Coupled mode theory describes coupling to ports:

$$\frac{d}{dt}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = i\mathcal{M}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + D^{\mathrm{T}}\begin{pmatrix} s_1^+ \\ s_2^+ \end{pmatrix} \qquad \begin{pmatrix} s_1^- \\ s_2^- \end{pmatrix} = C\begin{pmatrix} s_1^+ \\ s_2^+ \end{pmatrix} + D\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$



Suh, Wang, and Fan, IEEE J. Quant. Electron. 40, 1511 2004

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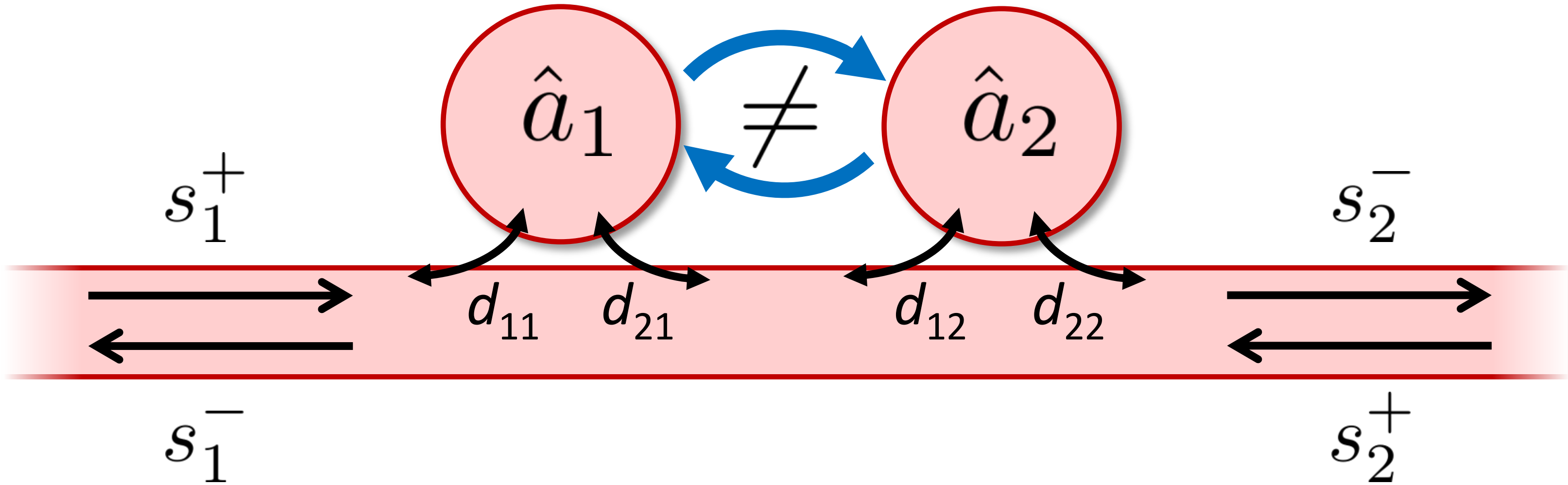
$$\begin{pmatrix} s_1^- \\ s_2^- \end{pmatrix} = C\begin{pmatrix} s_1^+ \\ s_2^+ \end{pmatrix} + D\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

yields scattering matrix:

$$\begin{pmatrix} s_1^- \\ s_2^- \end{pmatrix} = S\begin{pmatrix} s_1^+ \\ s_2^+ \end{pmatrix}$$

$$S = C + \mathrm{i}D(M+\omega I)^{-1}D^T$$



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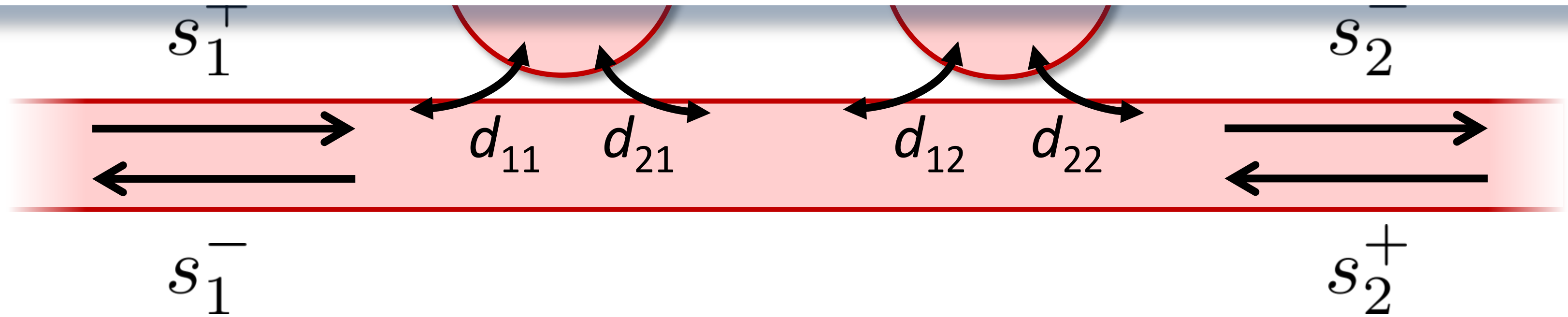
asymmetric optical system *nonreciprocal mode coupling*

$$S_{21} - S_{12} = i \det D \, \frac{(m_{12} - m_{21})}{\det(M + \omega I)}$$



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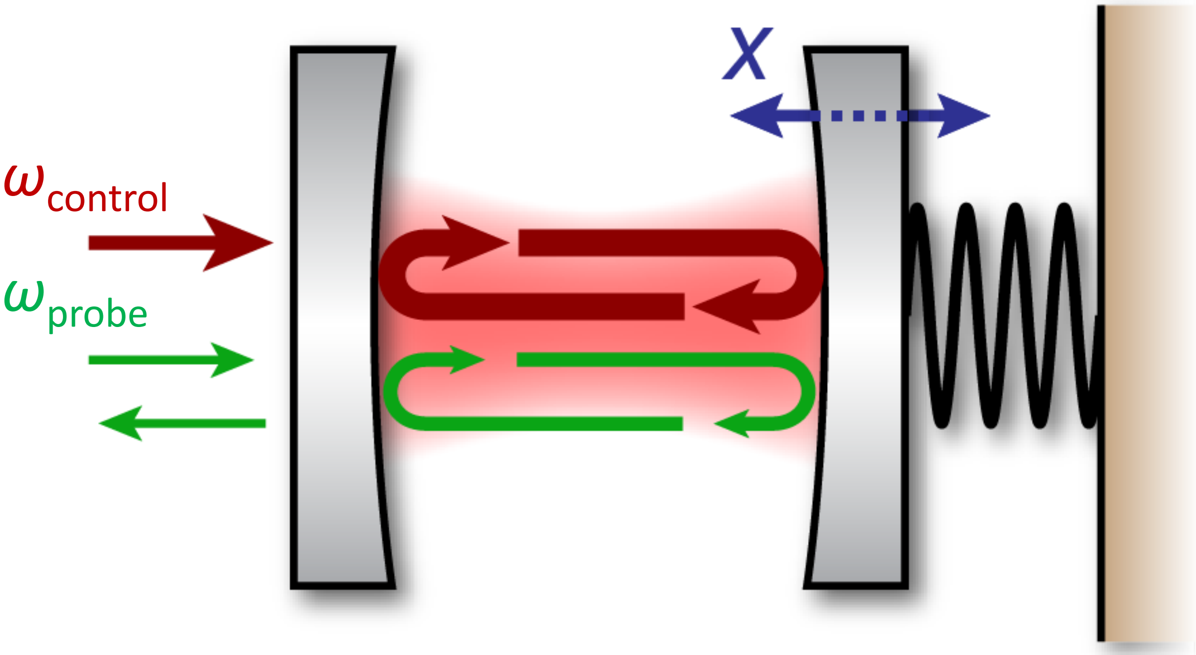
Cavity Optomechanics



Total optical field:

$\alpha + \hat{a}$

strong control — **weak probe**

Linear coupling between optical probe and mechanical resonator:

$$\hat{H}_{\mathrm{int}} = \hbar G x_{\mathrm{zpf}} (\alpha \hat{a}^\dagger \hat{b} + \alpha^* \hat{a} \hat{b}^\dagger)$$

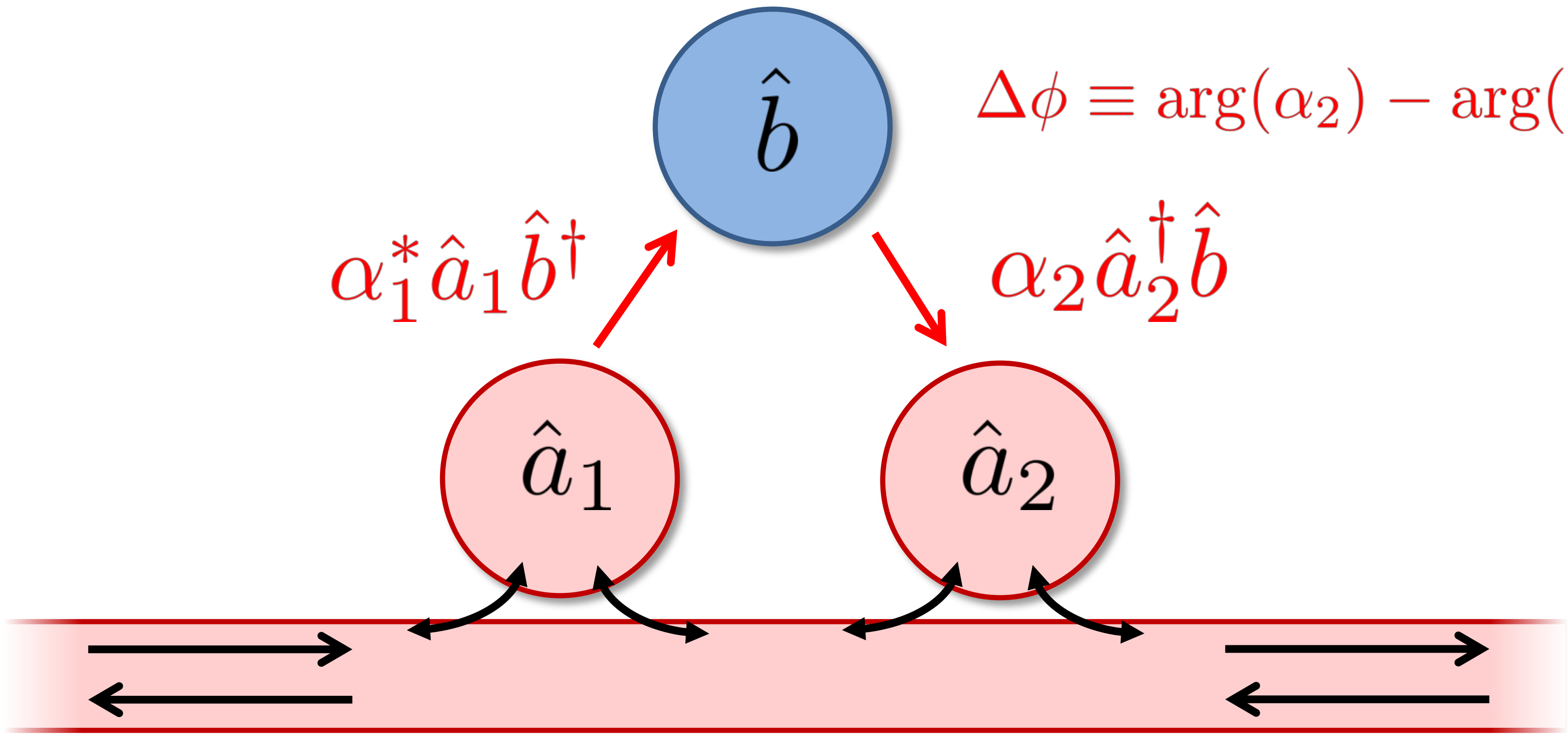
Nonreciprocal optomechanical phase

$$\hat{H}_{\mathrm{int}} = \hbar G x_{\mathrm{zpf}}(\alpha_1 \hat{a}_1^\dagger \hat{b} + \alpha_1^* \hat{a}_1 \hat{b}^\dagger + \alpha_2 \hat{a}_2^\dagger \hat{b} + \alpha_2^* \hat{a}_2 \hat{b}^\dagger)$$

Nonreciprocal optomechanical phase

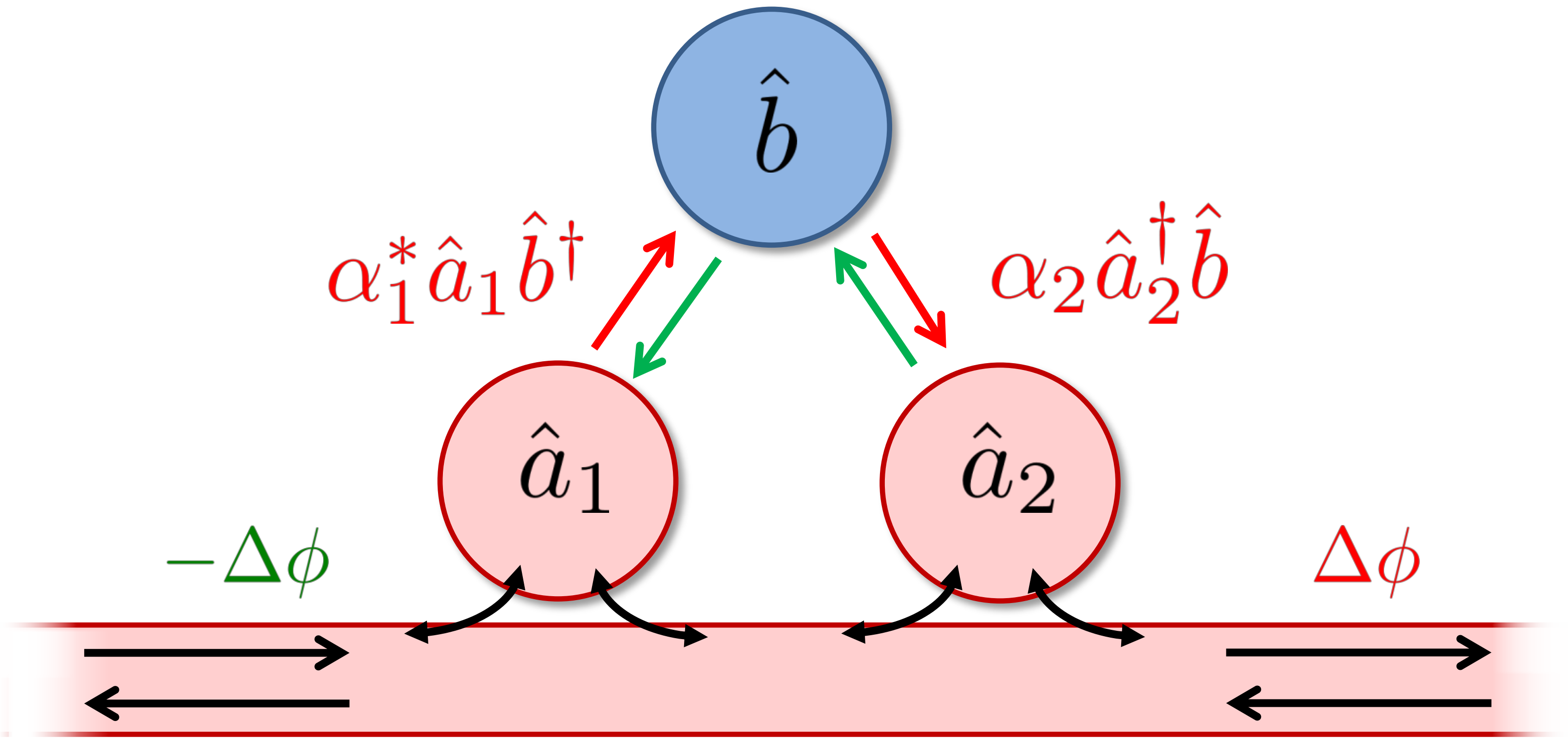
$$\hat{H}_{\mathrm{int}} = \hbar G x_{\mathrm{zpf}} (\alpha_1 \hat{a}_1^\dagger \hat{b} + \alpha_1^* \hat{a}_1 \hat{b}^\dagger + \alpha_2 \hat{a}_2^\dagger \hat{b} + \alpha_2^* \hat{a}_2 \hat{b}^\dagger)$$

$$\Delta\phi \equiv \arg(\alpha_2) - \arg(\alpha_1)$$



Nonreciprocal optomechanical phase

$$\hat{H}_{\rm int} = \hbar G x_{\rm zpf}(\alpha_1 \hat{a}_1^\dagger \hat{b} + \alpha_1^* \hat{a}_1 \hat{b}^\dagger + \alpha_2 \hat{a}_2^\dagger \hat{b} + \alpha_2^* \hat{a}_2 \hat{b}^\dagger)$$



Optimal nonreciprocity when two optical modes are driven at $\pi/2$

see: Habraken et al., New J. Phys. 14, 115004 (2012),
Xu and Li, Phys. Rev. A 91, 053854 (2015)

Optomechanical nonreciprocity

Optomechanics ⟷ **Coupled-Mode Theory**

<u>Equations of motions</u>

$$\frac{d}{dt}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \mathrm{i}\begin{pmatrix} \bar{\Delta}_1 + \mathrm{i}\kappa_1/2 & 0 \\ 0 & \bar{\Delta}_2 + \mathrm{i}\kappa_2/2 \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \mathrm{i}\begin{pmatrix} g_1(b+b^\dagger) \\ g_2(b+b^\dagger) \end{pmatrix} + D^{\mathrm{T}}\begin{pmatrix} s_1^+ \\ s_2^+ \end{pmatrix}$$

$$\frac{d}{dt}b = (-\mathrm{i}\Omega_m - \Gamma_m/2)\,b + \mathrm{i}(g_1^* a_1 + g_1 a_1^\dagger + g_2^* a_2 + g_2 a_2^\dagger)$$

Solve in Fourier Domain

Optomechanical nonreciprocity

Optomechanics ⟷ **Coupled-Mode Theory**

$$M + \omega I = \begin{pmatrix} \Sigma_{o_1} \mp \frac{|g_1|^2}{\Sigma_m^{\pm}} & \mp \frac{g_1 g_2^*}{\Sigma_m^{\pm}} \\ \mp \frac{g_1^* g_2}{\Sigma_m^{\pm}} & \Sigma_{o_2} \mp \frac{|g_2|^2}{\Sigma_m^{\pm}} \end{pmatrix}$$

$$S_{21} - S_{12} = \frac{\mathrm{i}\,\det(D)(m_{12} - m_{21})}{\det(M + \omega I)}$$

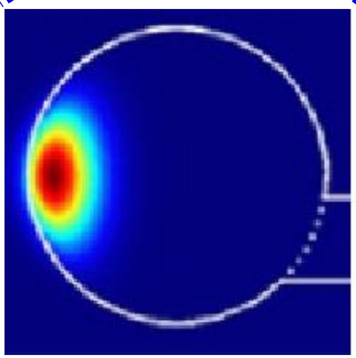
Importantly $m_{12} - m_{21} \propto \sin \Delta\phi$

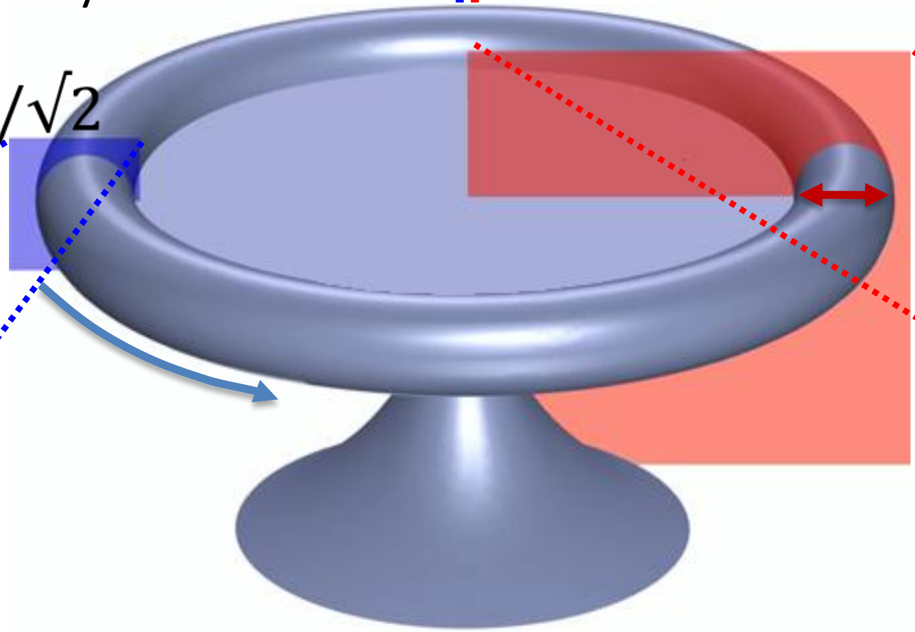
With $\Delta\phi$ the difference between drive phases

Microtoroidal optomechanical resonators

Optical whispering gallery mode

$$\hat{a}_{cw} = (\hat{a}_1 + i\hat{a}_2)/\sqrt{2}$$





Optical frequency: 200 THz
Optical Finesse: 10^5 - 10^6
Optical quality factor: 10^7 - 10^8

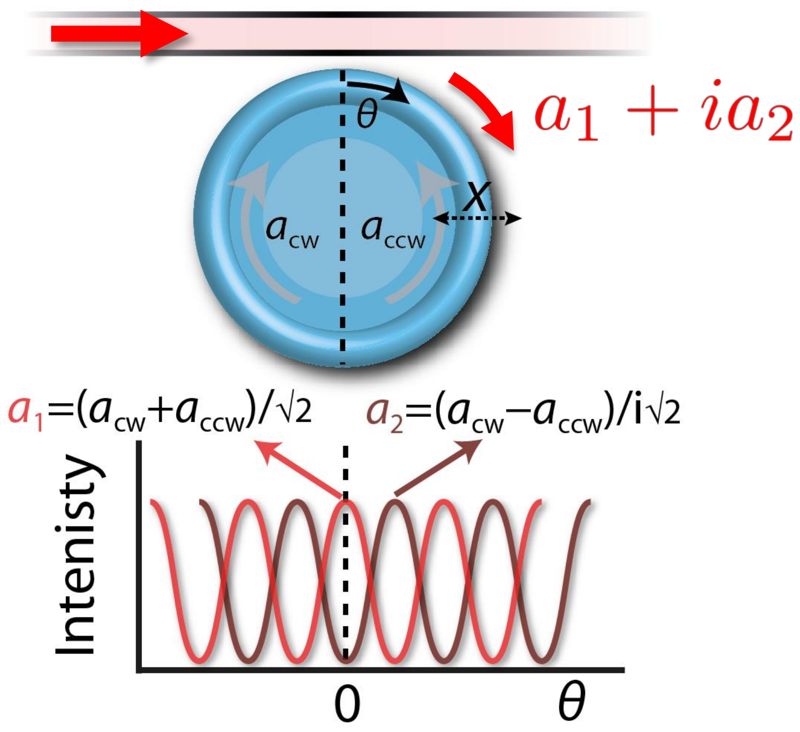
Mechanical radial breathing mode

Mechanical frequency: ~50 MHz
Mechanical quality factor: 10^3 - 10^4

Control field phase difference

- Even and odd optical modes (a_1, a_2) in ring; superpositions of rotating waves

- Even/odd basis preferred (reciprocity)

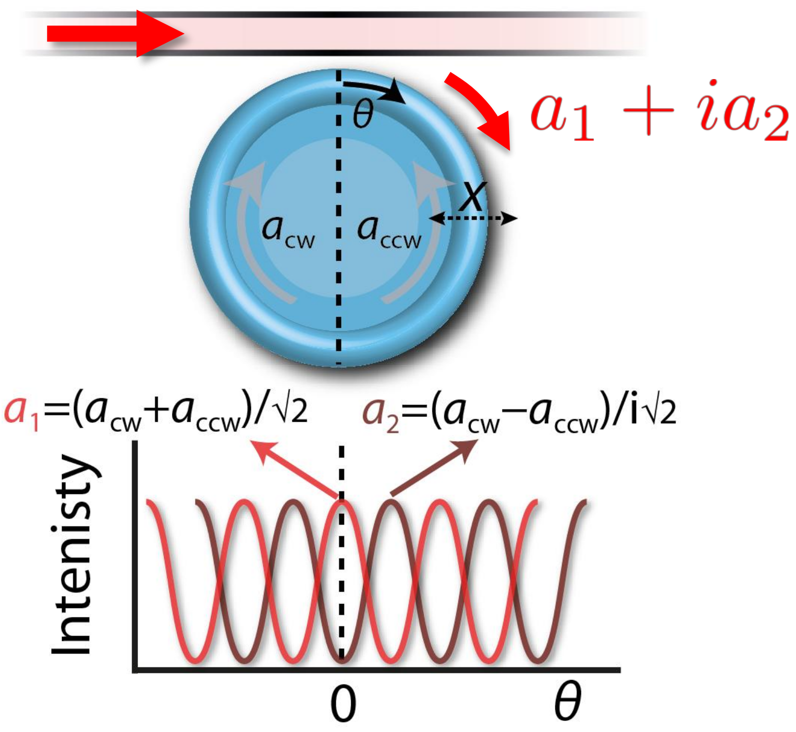


Control field phase difference

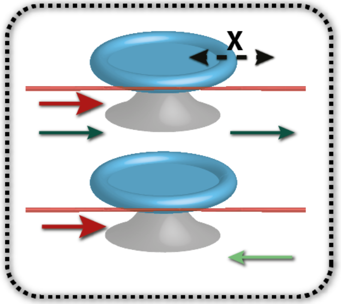
- Even and odd optical modes (a_1, a_2) in ring; superpositions of rotating waves

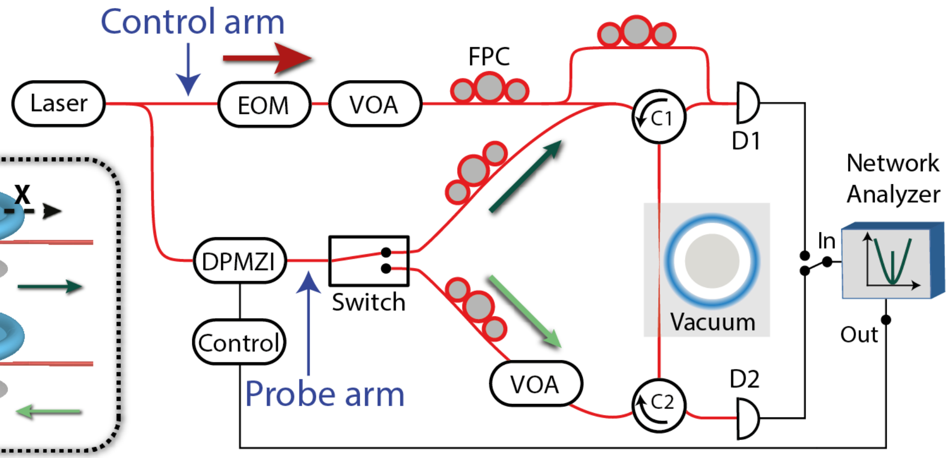
- Even/odd basis preferred (reciprocity)

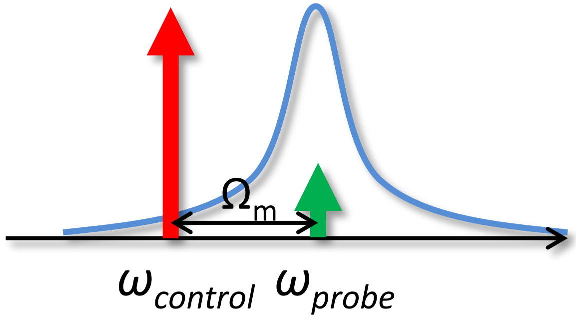
- $\pi/2$ control phase difference achieved by pumping from one side



Quantifying optomechanical nonreciprocity







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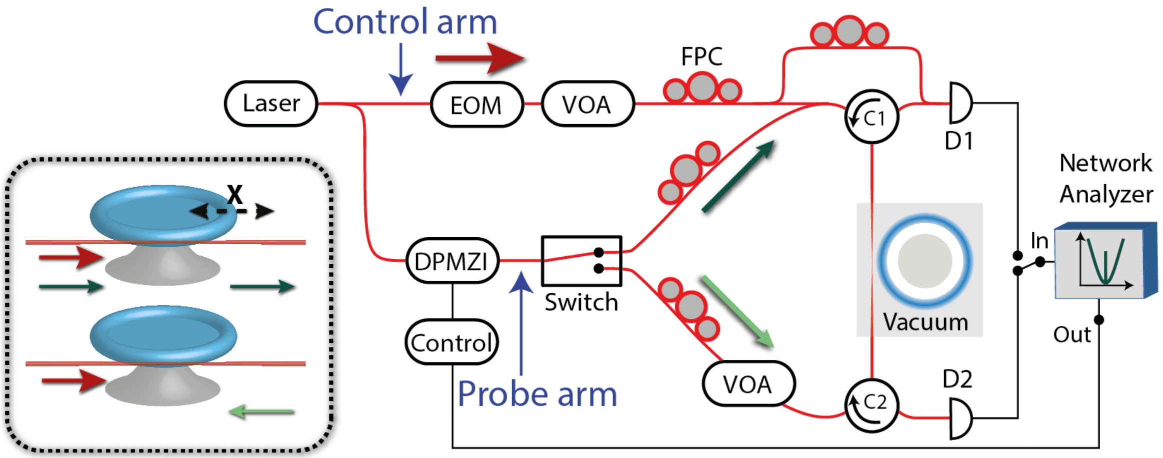
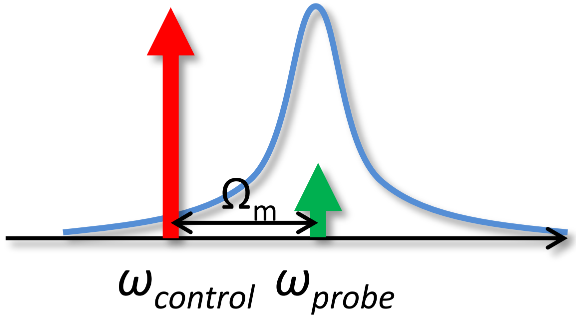
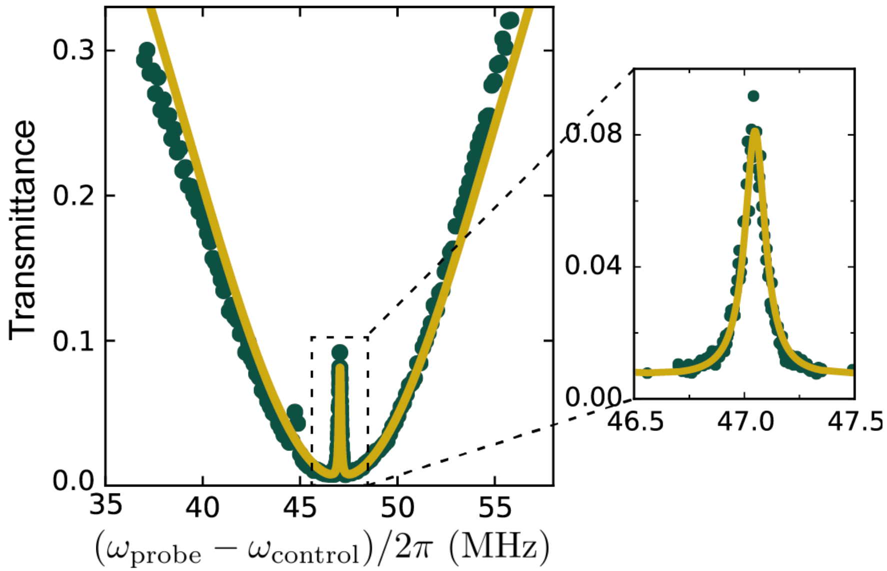


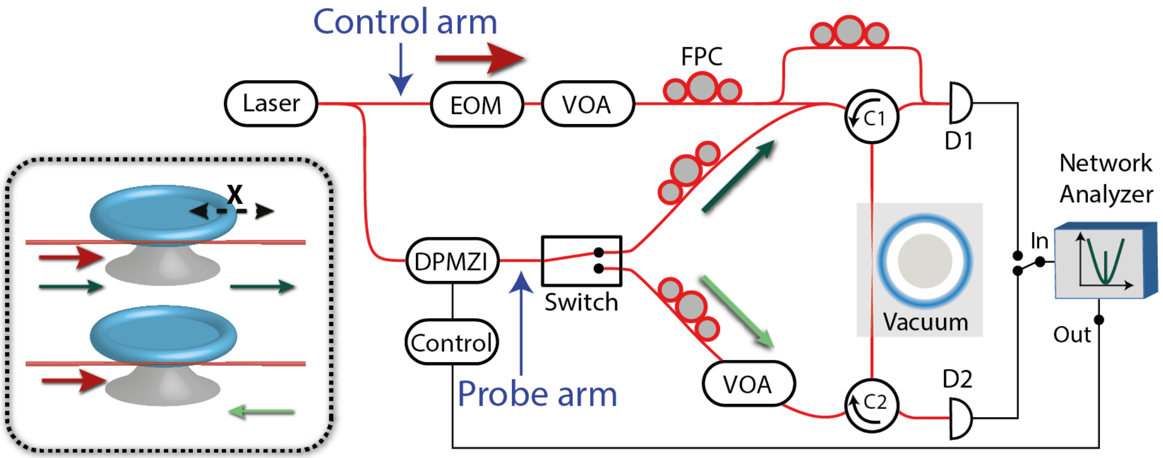


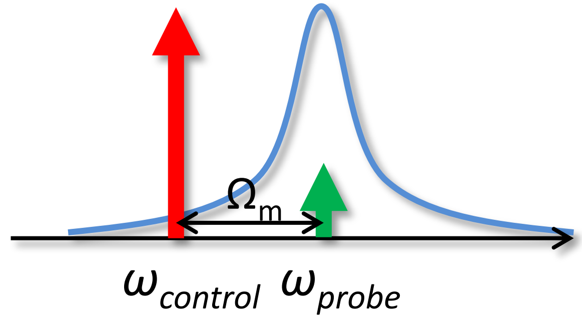


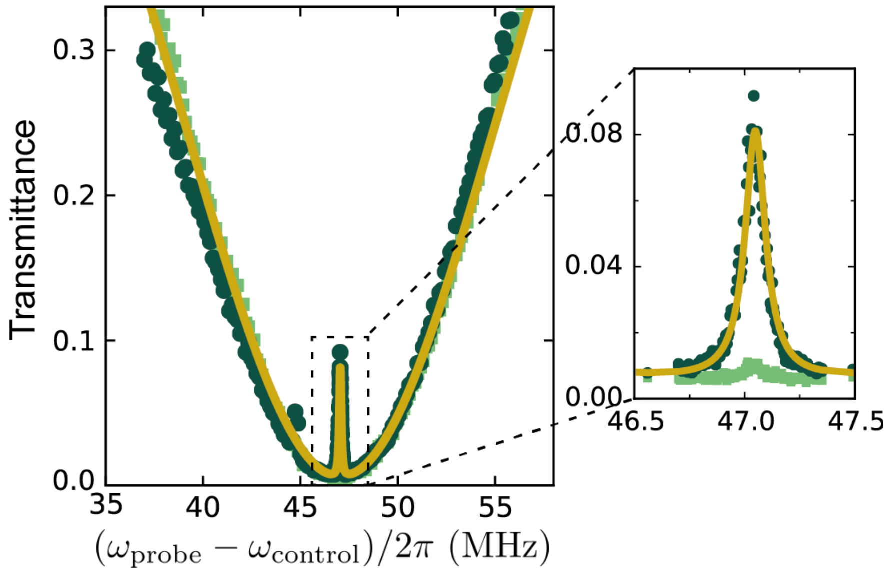
Optomechanically induced transparancy

Weis et al., Science 330, 1520 (2010)

Quantifying optomechanical nonreciprocity







Optomechanically induced transparancy

Weis et al., Science 330, 1520 (2010)

… can be nonreciprocal

proposal: Hafezi&Rabl, Opt. Express 20, 7672 (2012)

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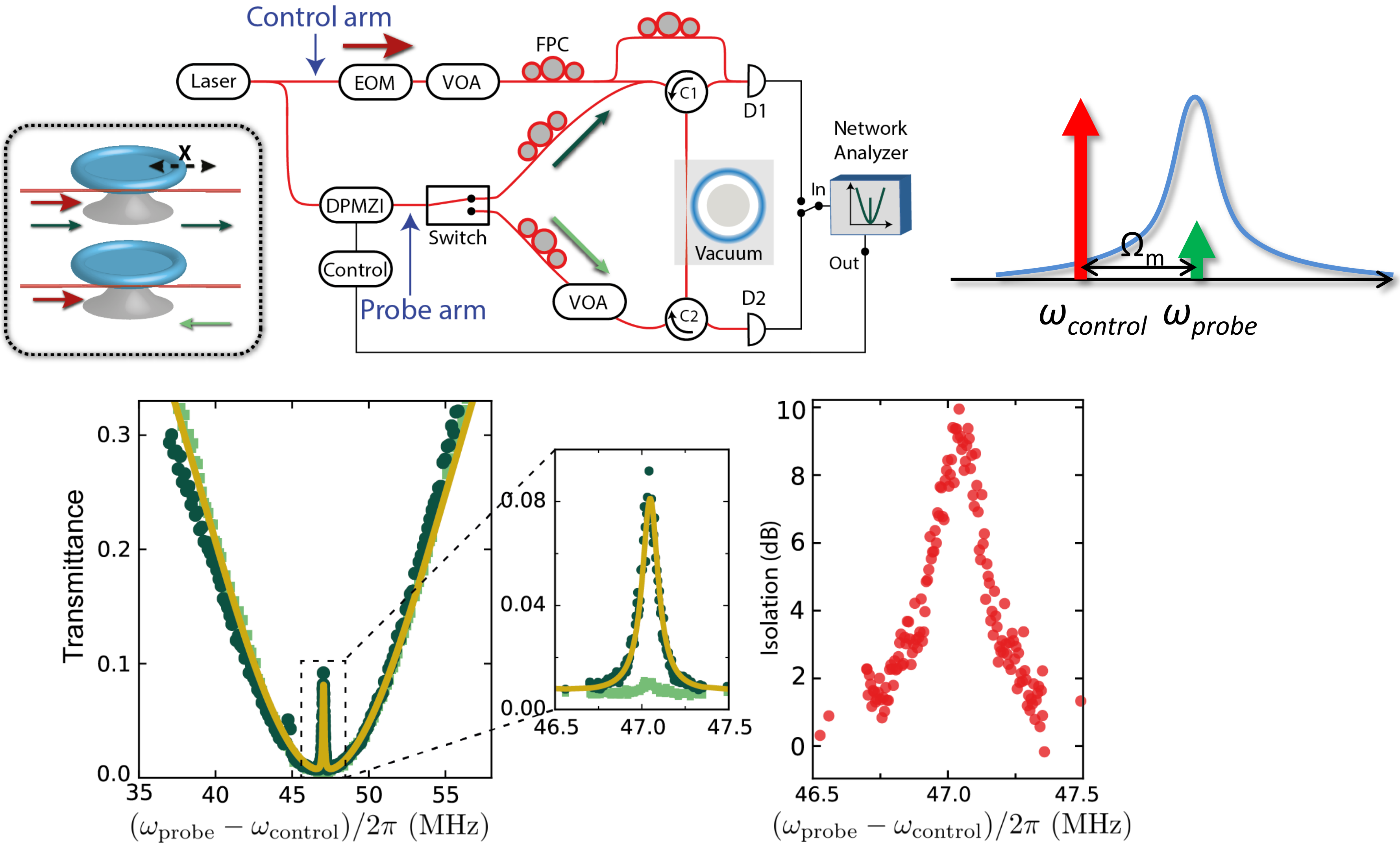
Quantifying optomechanical nonreciprocity



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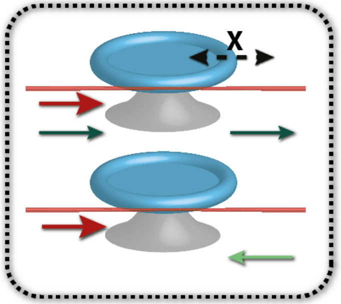
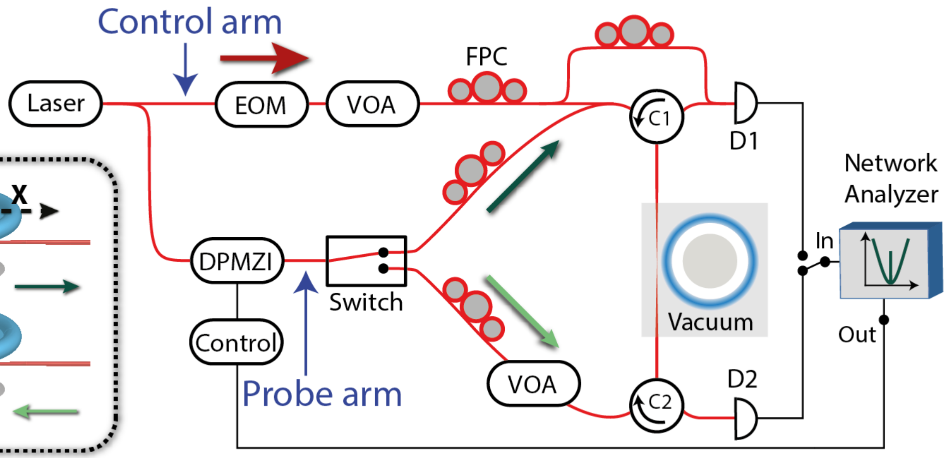
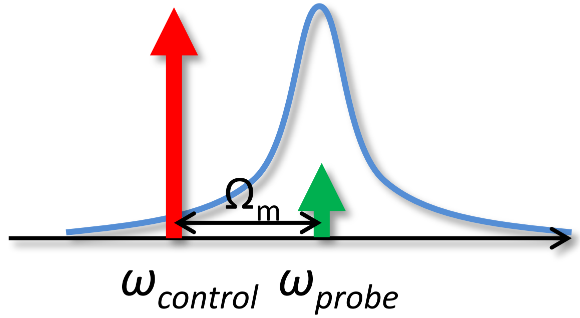
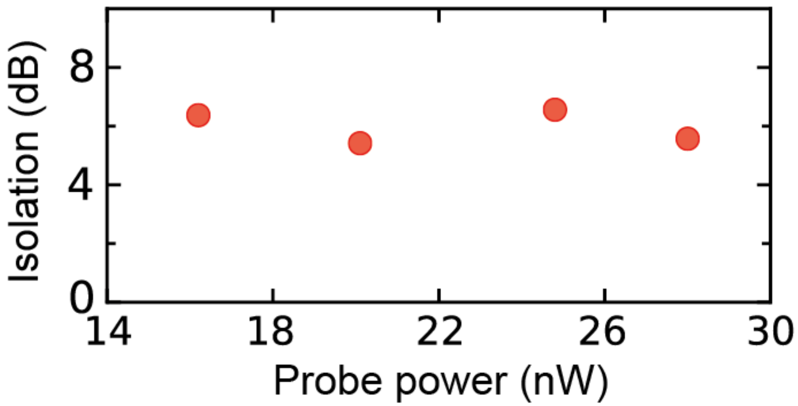
Isolation – Power dependence and bandwidth







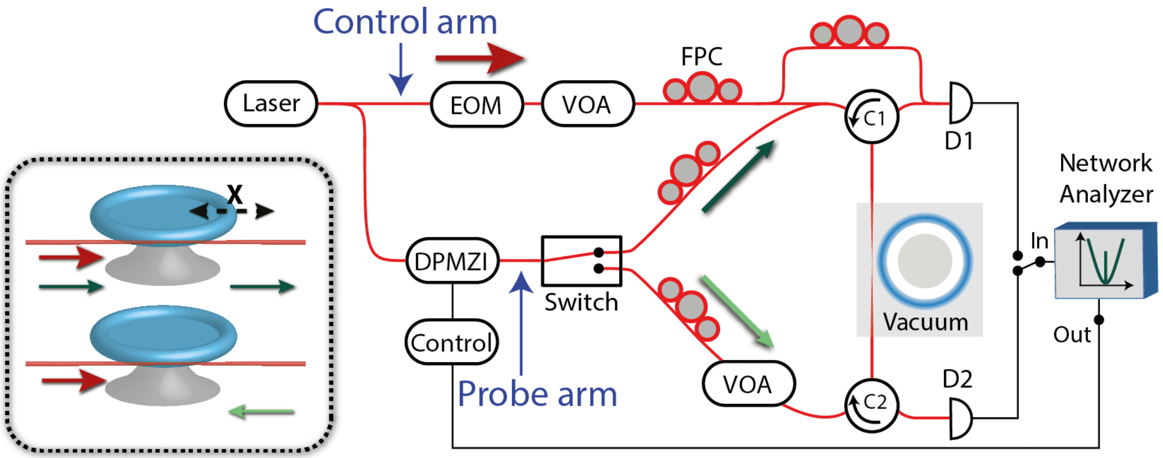


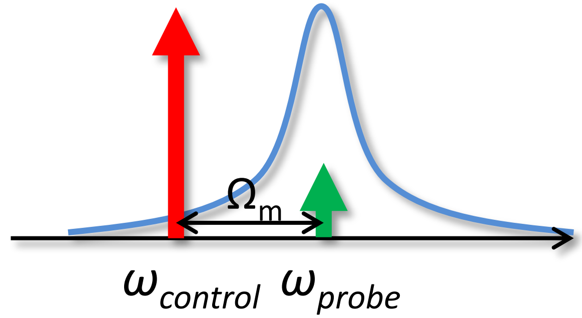
Isolation independent of probe power

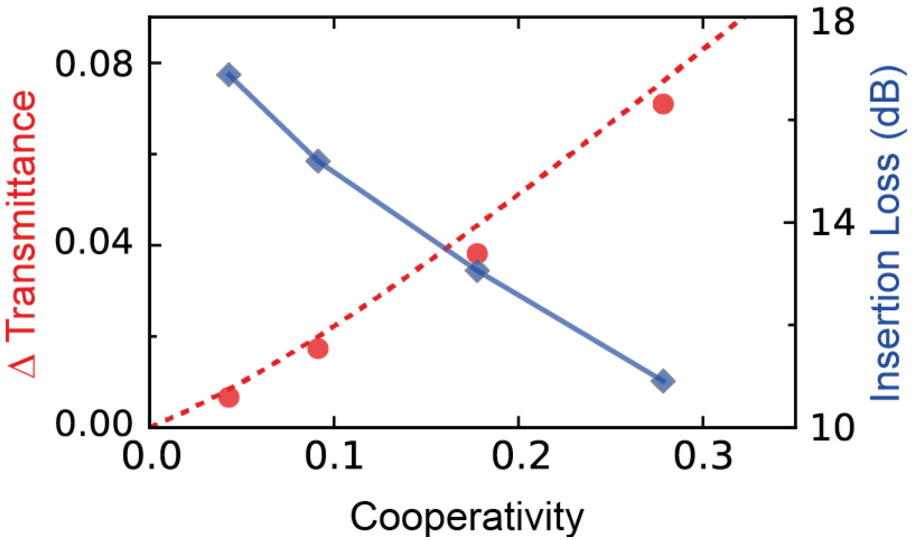
(unlike Kerr-based asymmetry)

e.g. Manipatruni, PRL 102, 213901 (2009), Fan, Science 335, 447 (2012)

Isolation – Power dependence and bandwidth







Cooperativity:

$$\mathcal{C} = \frac{4\, g_0^2}{\kappa \Gamma_m} \bar{\mathrm{n}}_{\mathrm{cav}}$$

Isolation:

$$\Delta \mathrm{T} = \frac{\mathcal{C}^2}{(\mathcal{C}+1)^2}$$

Bandwidth:

$$\Gamma_{\mathrm{eff}} = (1 + \mathcal{C})\Gamma_m$$

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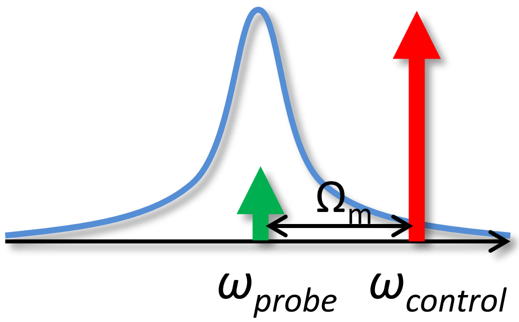
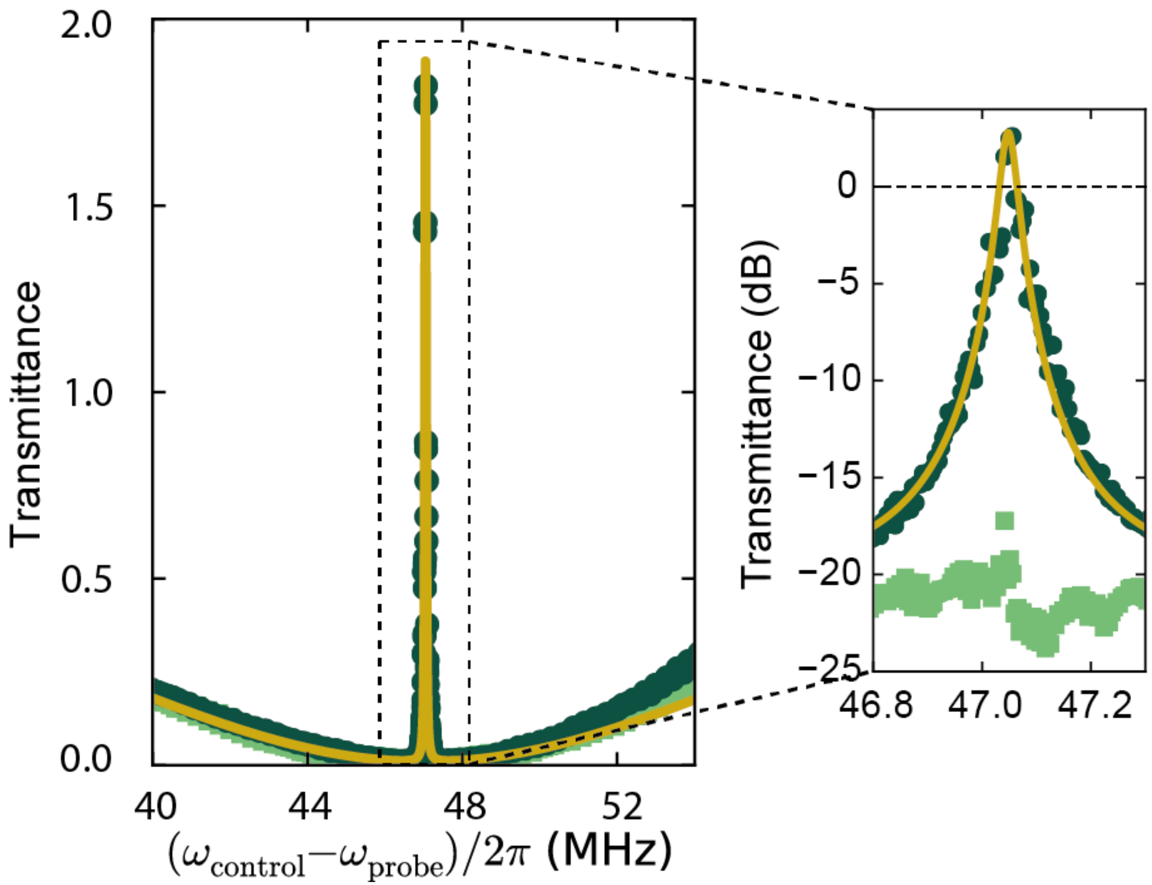
Nonreciprocal amplification





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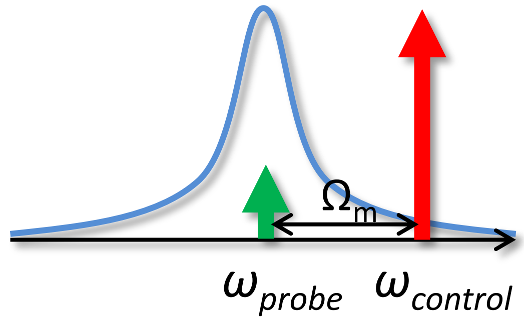
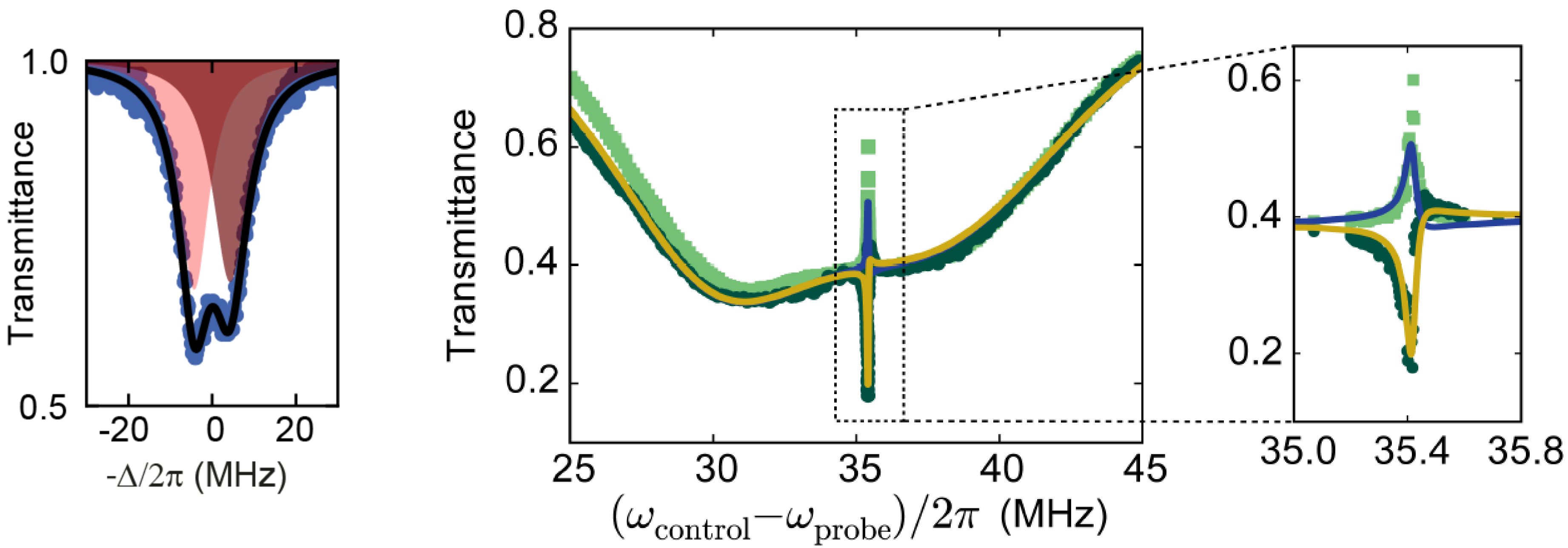
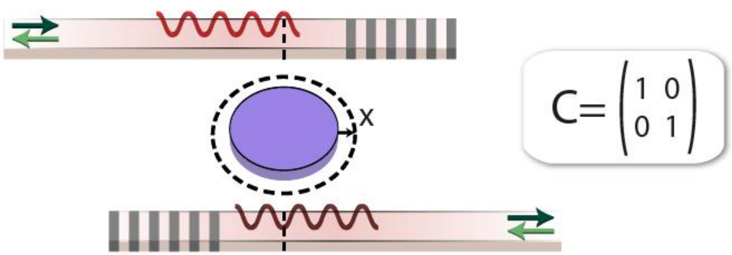
Nonreciprocity without optical degeneracy

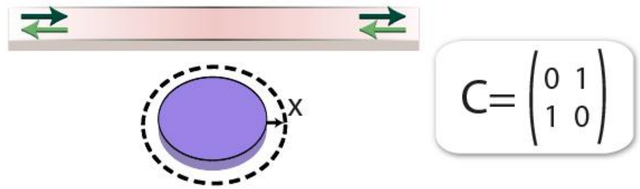




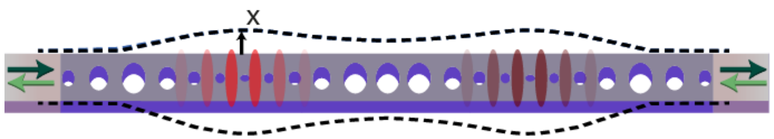
Outlook

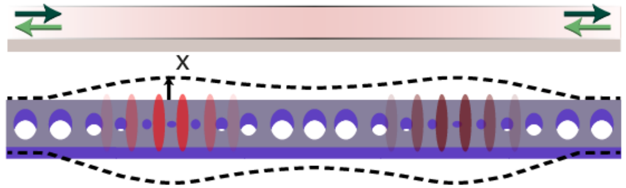
- Ring resonators



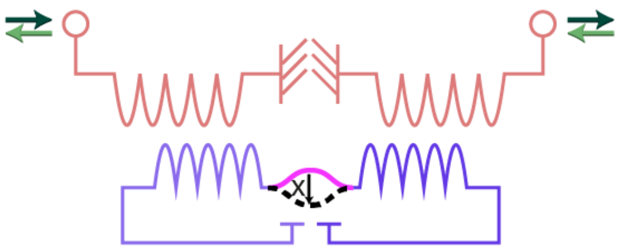


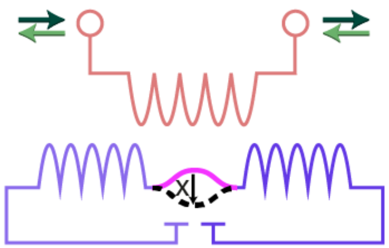
- Photonic crystals





- Superconducting circuits





F. Ruesink, M.-A. Miri, A. Alù, and E. Verhagen, *arXiv*: **1607.07180**

Conclusions

- Optimal nonreciprocity in two-mode (optomechanical) systems
- Crucial role of control field phase
- Demonstrated isolation in optomechanical ring resonator
- Building blocks of optomechanical metamaterials

$$S_{21} - S_{12} = i \det D \frac{(m_{12} - m_{12})}{\det(M + \omega I)}$$